A natural experiment in the impact of interest rates on beta

Syed Abuzar Moonis[*] Ajay Shah[†]

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[*]Part–time Ph.D student at IGIDR, Mumbai Address: A-168 Sector 26 Noida, India 201301. Phone +91-0118-4552370 email `<moonis@igidr.ac.in>`.

[†]IGIDR, Goregaon (E), Bombay 65, India. Phone +91-22-8400919 x557, email `<ajayshah@igidr.ac.in>`, and URL `http://www.igidr.ac.in/~ajayshah`

Abstract

The option-pricing approach to corporate finance offers a structural model explaining the beta of the equity of a firm. In this paper, we focus on testing one prediction of this approach: that an increase in interest rates should be associated with a decrease in betas. We utilise a unique natural experiment which took place in India, where changes in the stance of monetary policy give us two distinct years with a large difference of 584 basis points in the short-end interest rate. In our empirical strategy, we estimate time-varying betas using the modified Kalman filter, and form quartile portfolios based on the change in V/B to control for changes in leverage. We find that the theoretical prediction is largely successful.

JEL classification: G12

Keywords: Explaining firm-level betas, Option pricing, OPT-CAPM model, modified Kalman filter, natural experiment.

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1 Introduction

The option–pricing approach has been enormously influential in corporate finance. Using options reasoning, equity is a call option on the assets of the firm. This fact, coupled with option–pricing theory, has given us many new lines of attack on questions about firms. For example, in recent years, an entire literature has sprung up in the field of credit risk, which uses this option–pricing setup, and utilises information from the stock market for measuring the default probability of firms.

Galai & Masulis (1976) applied this approach to the determination of the beta of the firm. In this approach, there is a "core" beta, which is a characteristic of the activities of the firm in the real economy. The financial structure of the firm is an overlay upon this, which determines the beta of the equity of the firm. By applying option–pricing theory, in a simplified model, they obtain an elegant closed-form formula for the beta of the equity of a firm.

As of date, there has been little empirical testing of this "structural" model. DeJong & Collins (1985) study the behaviour of 100 firms in the United States over two periods. Interest rate volatility was much higher in the second period (January 1972 to December 1980) when compared with the first period (July 1962 to December 1971). They find that beta volatility was also higher in the second period. This helps us conclude that there is a relationship between interest rates and beta which is broadly consistent with the structural model.

We exploit a unique natural experiment which took place in India where a set of monetary policy measures changed the 90–day interest–rate by 725 basis points within a short period of 378 days. This natural experiment allows us to focus on two periods: Period I (1995–96) and Period II (1997–98) where the average 90–day interest rate differs by 584 basis points. This is a large change in the interest rate. We use this to test one aspect of the structural model: the prediction that $\frac{\partial \beta}{\partial r_F} < 0$.

The option–pricing model predicts that this reduction in interest rates should be accompanied by an increase in beta. However, we find that many firms deleveraged over this period, which has the opposite influence on beta. We control for this by forming quartiles by the change in leverage. Once this is done, we find that the theoretical predictions about the change in beta are borne out quite clearly. The quartile portfolio where leverage rose shows the strongest increase in beta, and there is a clear pattern in the change in beta as we go down to the other three quartile portfolios.

Our results broadly validate the predictions about the relationship between interest rates and beta, made by the highly simplified model based on option–pricing theory.

The remainder of this paper is organised as follows. In Section 2, we summarise the model which exploits option–pricing theory to give us a structural model of the beta of firm equity. Section 3 describes the natural experiment which took place with monetary policy and interest rates in India from 1995 till 1998. Section 4 describes the testing strategy that we employ. Section 5 shows the results, and Section 6 concludes.

2 Structural modeling of beta

Along with the first derivation of their option–pricing formula, Black and Scholes offered a fundamental insight into the financial structure of the limited liability firm (Black & Scholes 1973). They pointed out that equity holders have a call option on the value of the firm, and bond holders are short a put option. In this approach, the value of equity is the value of a call option on the assets of the firm.

In the simplest case, we assume that interest–rates are non-stochastic and the debt of the firm is a single bond maturing on T. Here, the market capitalisation of equity (E) is the value of a call option on the assets of the firm (V) with the strike price X set to the face value of debt (B):

$$
\begin{aligned}
X &= B && (1)\\
E &= VN(d_1) - Xe^{-r_F T}N(d_2) && (2)\\
\text{where } d_1 &= \left(\ln\frac{V}{X} + r_F + \frac{\sigma_V^2 T}{2}\right)(\sigma_V\sqrt{T})^{-1} && (3)\\
\text{and } d_2 &= d_1 - \sigma_V\sqrt{T} && (4)
\end{aligned}
$$

The option-pricing approach has had important consequences in many areas of corporate finance. For example, starting with Merton (1974), this approach has been used in harnessing equity market information for the measurement of credit risk of firms.

Galai & Masulis (1976) extended this idea to offer a structural model explaining beta at the firm level. They use the idea that the beta of the assets of the firm, β_V, is a characteristic of the real activities of the firm. They treat β_V as exogenously given. The financial structure of the firm determines the beta of the equity of the firm, i.e. β_E. This relationship can be derived from Equation 2:

$$\beta_E = N(d_1)\frac{V}{E}\beta_V \tag{5}$$

Substituting for E gives us a closed-form expression for β_E :

$$\beta_E = \frac{VN(d_1)}{VN(d_1) - Be^{-r_F T}N(d_2)}\beta_V \tag{6}$$

This equation can be used to obtain comparative statics of β_E. Specifically, differentiating in r_F, we get $\frac{\partial \beta_E}{\partial r_F} < 0$. Appendix A offers derivations of all these results.

Thus, the application of option pricing theory to corporate finance, in this simplified setting, predicts that stock betas should drop with a small increase in interest rates. In this paper, we focus on testing this prediction.

DeJong & Collins (1985) dealt with a related aspect of the relationship between interest rates and beta. They test whether higher interest–rate volatility is associated with higher beta volatility. They find that this prediction is, indeed, borne out in their dataset. However, their focus is on the *volatility* of betas and interest rates, and not on the direction of this relationship.

This structural model explaining β_E is also interesting from the perspective of the substantial empirical literature on beta instability (Wells 1996). This extensive literature has rejected the null of beta stationarity in many countries and time–periods. The structural model for β_E helps us better understand the sources of beta instability and go beyond purely reduced form analysis of beta.

2.1 Confounding effects

While this theoretical setup gives us a prediction for the direction of the relationship between interest rates and beta, interest rates are not the only source of beta variability. Equation 6 yields four other comparative static results[1]:

[1]See Galai & Masulis (1976) for detailed proofs.

Beta is a decreasing function of the value of the firm, V.	$\frac{\partial \beta_E}{\partial V} < 0$
Beta is a increasing function of the face value of firm's debt, B.	$\frac{\partial \beta_E}{\partial B} > 0$
Beta is a decreasing function of the volatility of firm value, σ_V.	$\frac{\partial \beta_E}{\partial \sigma_V} < 0$
Beta is a decreasing function of the time to maturity of debt, T.	$\frac{\partial \beta_E}{\partial T} < 0$

Thus we have four variables which are firm–characteristics (V, B, σ_V, T) and one macro-economic variable (r_F) influencing β_E.

2.2 V and σ_V are unobserved

Two of the variables affecting β_E – the value of the firm V and its volatility σ_V – are not directly observable. However, these can be inferred using observables, as follows. Applying Ito's lemma to the call option formula, we obtain:

$$\sigma_E = \sigma_V \frac{V}{E} N(d_1) \tag{7}$$

Equation 7 and Equation 2 can be solved for two unknowns - σ_V and V - using the observed values for E and σ_E.

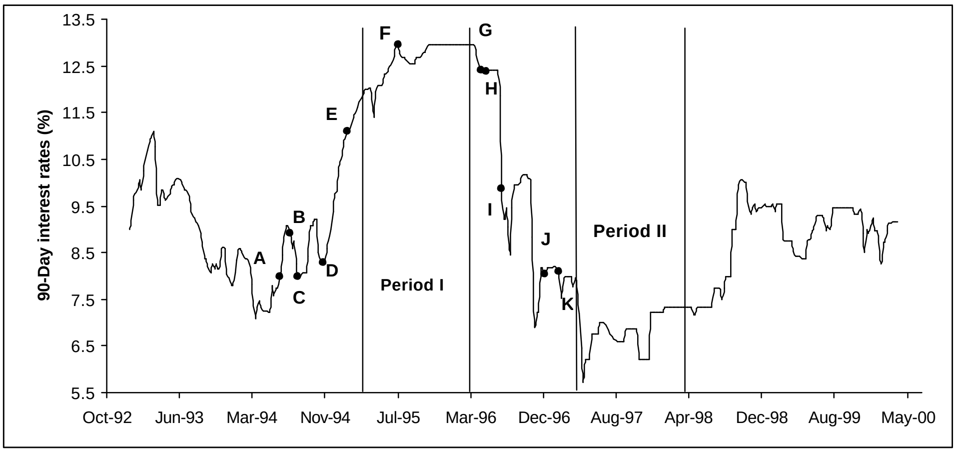
3 A natural experiment

We seek to learn about the relationship between interest rates and stock betas. Under normal circumstances, stock betas evolve in response to changes in all five explanatory variables. Specifically, changes in the debt/equity mix, and in the maturity structure of the debt, influence the stock beta. Hence, under normal circumstances, it is relatively difficult to disentangle the impact of interest rates on beta.

In the decade of the 1990s, we observed an interesting natural experiment in India, where monetary policy was used to sharply increase interest rates, and then to sharply reduce interest rates. This is highlighted in Figure 1, which shows the time–series of the interest rate on the 90–day treasury bill.

Monetary tightening On 11 June 1994, the central bank, the Reserve Bank of India (RBI), embarked on a series of policy measures designed to tighten

Figure 1 The 90–day treasury bill, 1992–2000



Monetary tightening

A (11-Jun-1994) Cash Reserve Ratio (CRR) was raised from 14% to 14.5%.

B (09-Jul-1994) CRR was raised to 14.75%.

C (06-Aug-1994) CRR was raised to 15%.

D (29-Oct-1994) CRR for Foreign Currency Non-Resident (FCNR) Accounts was raised from 0% to 7.5%.

E (21-Jan-1995) CRR for Non-Resident accounts raised from 0% to 7.5%, and CRR for FCNR accounts was raised to 15%.

F (17-Jul-1995) Conditions for overdraft facility to stock brokers to draw money from banks were made more stringent.

Monetary easing

G (27-Apr-1996) CRR was reduced to 13.5%.

H (11-May-1996) CRR was reduced to 13%.

I (06-Jul-1996) CRR was reduced to 12%.

J (31-Nov-1996) CRR was reduced to 11%.

K (11-Jan-1997) CRR was reduced to 10.5%.

monetary policy. This restrictive monetary policy regime was motivated by a resurgence of double-digit inflation and monetary expansion caused by foreign capital inflows.

The figure shows restrictive policy measures from A till F, which brought about a sharp increase in interest rates. The Cash Reserve Ratio (CRR) that banks are required to hold was an important tool that was employed. Apart from the specific policy decisions shown in the future, RBI also increased interest rates on export credit. As part of the strengthening of prudential regulation of banks, March 1996 was the date by which banks had to attain a minimum capital-to-risk-weighted-assets ratio of 8%.

In response to these policy measures, the interest rate on the 90–day treasury bill rose sharply from 7.08% on 18-Mar-1994 to 12.96% on 11-Mar-95.

Monetary easing These restrictive measures were relaxed starting from the first quarter of 1996–97. The CRR was reduced from 14% on 27 April 1996 to 10.5% on 11 January 1997. The withdrawal of the tight monetary policy manifested in a sharp fall in interest rates during the second half of 1996 and early 1997.

The changing policy goals of RBI, and consequently the changing stance of monetary policy, are well documented in speeches by the RBI Governor over this period, and in annual reports of the RBI.

Figure 1 shows the time-series of the 90–day treasury bill interest rate from 1992 till 2000. We see that the highest interest rates, and the lowest interest rates, of this period are associated with this natural experiment. The difference between the highest and lowest interest rates seen here, of roughly 725 basis points within a short period of 378 days, is an extremely large change in interest rates. This suggests that the change in interest rate seen here was unusual when compared with "normal" interest rate fluctuations.

Thus, it appears that the high interest rates in 1995 and 1996 were inspired by exogenous policy shocks, as were the low interest rates in 1997 and 1998. Over this period, we had extremely large changes in the interest rate, while the other determinants of beta in the structural model may not have changed substantially. This suggests that we can learn something about the relationship between interest rates and beta by observing the response of firm betas to these events.

4 Testing procedure

Normally, tests of the relationship between r_F and β_E are inhibited by the presence of confounding factors. The structural model itself is nonlinear and an OLS regression, where all five explanatory variables are present on the right hand side, is of limited usefulness.

Table 1 Interest rates in the two periods

The table gives the summary of 90-day Treasury bill interest rate behavior across the two periods

	Period 1 1-Apr-1995 to 31-Mar-1996	Period 2 1-Apr-1997 to 31-Mar-1998
Minimum	11.40	5.72
Average	12.67	6.83
Maximum	12.97	7.33
St. Dev	0.38	0.40

In our setting, we have a remarkable situation where there was a large shock to one variable, r_F. In the entire history of interest rates in India, we do not have a comparable situation where there was such a sharp change in interest rates over such a short period. Over these years, the confounding factors would indeed display changes. The changes in beta caused by the confounding factors might be small enough, when compared with the change in beta caused by the change in r_F, to give us statistical efficiency in testing the impact of r_F on beta.

We may compare our situation against an ideal experimental design. In the ideal experiment, we would have N pairs of identical firms. One set of N firms would be placed in a low interest–rate regime, and the twin set of N firms would be placed in a high interest–rate regime. In this experiment, all confounding factors would be held constant, and the impact of r_F would be isolated.

The natural experiment that has been offered to us is not this ideal experiment. We do not observe two sets of identical firms; instead we observe the same set of firms across two periods, where the confounding factors have not been held constant. However, we can argue that the change in interest rates in our natural experiment is large, while the normal fluctuations of confounding factors are small. The effect of interest rates alone leads to the prediction that betas in period 2 should be lower than those in period 1.

In our statistical procedures, we go beyond a simple comparison of betas across periods, to directly address changes in firm leverage, which is one important source through which firm betas fluctuate.

4.1 Choice of periods for empirical work

Firms in India report accounting results for a financial year that normally runs from 1 April till 31 March. This suggests that we should focus on two periods for our comparison, defined as follows. The first period, the regime with high interest rates, runs from 1 April 1995 till 31 March 1996. The second period, the regime with low interest rates, runs from 1 April 1997 till 31 March 1998.

The intervening year, from 1 April 1996 to 31 March 1997, was an intermediate period between a high interest rate regime to a low interest rate regime, and it is not possible to unambiguously place it in either regime.

Table 1 shows summary statistics about interest rates in these two periods. We see that the mean interest rate in Period 2 was 584 basis points lower than that seen in Period 1.

4.2 Testing for the change in beta

We seek to test the hypothesis that the beta of a stock is an increasing function of the risk–free interest rate. This is done using the following model with time–varying betas –

$$r_{it} = \beta_{it} r_{Mt} + \epsilon_{it} \quad (8)$$
$$\beta_{it} = \bar{\beta}_{ik} + \phi(\beta_{it-1} - \bar{\beta}_{ik}) + \eta_{it} \quad (9)$$

Here r_{it} is the return on stock i on day t, β_{it} is the beta of stock i on day t, r_{Mt} is the return on the index on day t, ϵ_{it} is the market model residual for stock i on day t, $\bar{\beta}_{ik}$ is the mean beta for stock i in regime k, η_{it} is shock on the beta process for stock i on date t.

We use the convention $k = 1$ in the first period, when $t \in$ (01/04/1995 - 31/03/1996), $k = 2$ in the second period when $t \in$ (01/04/1997-31/03/1998), $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and $\eta_{it} \sim N(0, \sigma_\eta^2)$. The model allows for a discrete shift in the mean beta, $\bar{\beta}_{ik}$, between the two time periods.

Our use of the AR(1) model for the time–varying beta process reflects the empirical evidence in the beta instability literature (Ohlson & Rosenberg 1982). It is also consistent with previous research using Indian data (Moonis & Shah 2000), where time–varying betas proved to follow the AR(1) process or a random coefficients model, both of which are nested in the above specification.

We assume that the market model residual ϵ_{it} follows a GARCH(1, 1) model. The GARCH(1, 1) structure in market model variance is consistent with the evidence of volatility clustering in financial time series (Bollerslev et al. 1992).

Equations 8–9 are estimated using the modified Kalman Filter of Harvey et al. (1992).

Table 2 Tests for change in beta at the stock level

This table shows the number of firms, out of the dataset of 92 firms, where H_0 is rejected. For example, the 95% test with $H_0 : \beta_{i1} = \beta_{i2}$ against $H_1 : \beta_{i1} < \beta_{i2}$ is rejected for 6 of the 92 firms.

	Number of firms where H_0 is rejected		
H_1	2.5%	5%	10%
$\beta_{i2} > \beta_{i1}$	14	23	33
$\beta_{i1} < \beta_{i2}$	4	6	8

4.3 Data description

The most liquid 100 firms in India are the constituents of the S&P CNX Nifty and Nifty junior indexes (Shah & Thomas 1998). Some of these firms did not exist in both Period 1 and Period 2. Hence we are reduced to a dataset of 92 firms. We use the NSE-50 index as the market index.

5 Results

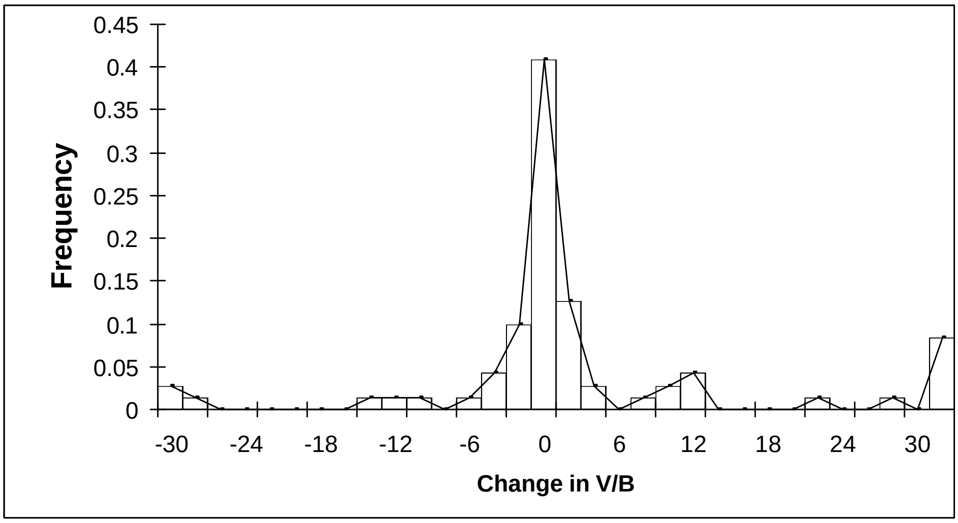
5.1 Test for individual firms

To test the effect of an interest rate change from Period I to Period II on betas of individual firms, we estimate the model given in Equations 8-9, for all the firms for the two periods. Using these estimates, we find that of the 92 firms, in 60 cases, $\bar{\beta}_{i2}$ was found to be higher that $\bar{\beta}_{i1}$.

We test the null hypothesis $H_0 : \bar{\beta}_{i2} = \bar{\beta}_{i1}$ against the alternative $H_1 : \bar{\beta}_{i2} > \bar{\beta}_{i1}$. Table 2 shows the results of these tests. The null hypothesis of $H_0 : \bar{\beta}_{i2} = \bar{\beta}_{i1}$ was rejected for 14, 23, 33 firms at the 2.5%, 5%, and 10% significance levels, respectively against the alternative of $H_1 : \bar{\beta}_{i2} > \bar{\beta}_{i1}$. In comparison, we could reject H_0 or 4, 6, and 8 firms at 2.5%, 5% and 10% significance level for the alternative hypothesis $H_1 : \bar{\beta}_{i2} < \bar{\beta}_{i1}$.

These results suggest that beta was higher for most firms in Period II. There are, however, 32 firms for which the numerical value of beta in Period II was lower than that in Period I. For 8 of these firms, the fall in beta in Period II was significant at a 10% level of significance. In order to understand this better, we need to control for confounding factors.

Figure 2 Distribution of the change in V/B amongst the 92 firms



5.2 Controlling for changes in leverage

Our ability to discern the impact of interest rates on beta is contaminated by the fact that Period I and Period II are two years apart. Over these two years, some of the confounding factors (at the firm level) could have changed. We will focus on leverage, which is particularly important as a channel through which actions of the firm can impact on beta.[2]

We use V/B as a measure of leverage : firms with low debt would have high values of V/B. If V/B changed from Period I to Period II, it would constitute a confounding factor which impacted on beta. Specifically, an decrease in leverage would be associated with a decrease in beta, which is in the opposite direction when compared with the impact of the reduction in r_F.

We compute the change in V/B in Period II when compared with that found in Period I. Figure 2 shows the distribution of the change in V/B. This shows that there is substantial cross-sectional variation in the change in V/B. Hence, we need to control for this when interpreting the change in beta.

[2]The volatility of the unlevered firm, σ_V, would change in response to changes in the nature of business of the firm. To the extent that firms do not greatly change their product mix over a period of two years, this is likely to be stable. The maturity pattern of the firms debt, T, is relatively unlikely to change sharply over a two year period, particularly given the lack of liquidity of the corporate bond market in India, which inhibits sharp changes in corporate financial structure on the part of the firm. Hence, we focus on changes in V and B, which enters our equations through the leverage V/B. CONFIRM THIS.

Table 3 Summary statistics about $\Delta\frac{V}{B}$ for quartile portfolios

This table shows summary statistics about the change in leverage for the quartile portfolios. Portfolio HIGH is the firms where the sharpest increase in V/B was found. The table shows that in this quartile, V/B for the median firm rose by 11.08.

	Change in $\Delta\frac{V}{B}$			
	Portfolio LOW	Portfolio 2	Portfolio 3	Portfolio HIGH
Minimum	-856.74	-1.93	-0.54	1.39
Median	-4.12	-1.05	-0.01	11.08
Maximum	-1.94	-0.54	1.105	-
Average	-55.58	-1.12	0.067	-

In order to deal with this problem, we sort firms by the change in V/B, and form four quartile portfolios. We label these portfolios as "LOW", "2", "3" and "HIGH" where HIGH is the portfolio of firms where the strongest deleveraging took place. We will measure the average change in beta seen across these quartiles.

Characteristics of the four quartile portfolios are shown in Table 3. In the quartile HIGH, leverage went down. In quartile 3, leverage seems to have been unchanged. In quartile LOW and quartile 2, leverage went up.

In our theoretical model, an increase in leverage is associated with an increase in beta. Hence, in the case of the LOW portfolio, the impact of the change in interest–rates and the change in leverage are in the same direction. We would expect a sharp increase in betas for LOW portfolio. The remaining three portfolios have opposing directions of influence: the reduction in leverage would be associated with a drop in beta while the reduction in interest rates would have the opposite effect. The confounding effect would be strongest for the HIGH portfolio, where we see a sharp reduction in leverage.

5.3 Other sources of confounding changes

The procedure of forming quartiles by the change in V/B controls for the confounding effect in changes to leverage. This leaves us with two sources of confounding effects: changes in T and changes in σ_V.

This procedure of forming quartile portfolios will give us an element of diversification in cancelling out the changes in beta that are caused by changes in σ_V. In each of the quartile portfolios, some firms will have changed their business profile in a way which increases σ_V while others may have reduced σ_V. By analysing the change in beta for portfolios, our results may be relatively uncontaminated by changes in σ_V.

Table 4 Change in σ_V across the portfolios

	Median σ_V	
	Period I	Period II
LOW	0.52	0.49
2	0.41	0.33
3	0.38	0.40
HIGH	0.55	0.60

Table 5 Results of interest rate effect on portfolio betas

This table shows the change in beta, and the change in V/B, for the quartile portfolios. The t-statistic for the change in portfolio beta, and its prob value, are also shown.

	$\bar{\beta}_{p2} - \bar{\beta}_{p1}$	$\Delta\frac{V}{B}$	$t_{\Delta\beta}$	$P(\bar{\beta}_{p2} > \bar{\beta}_{p1})$
Portfolio LOW	0.253	-4.127	2.192	0.014
Portfolio 2	0.151	-1.055	1.071	0.141
Portfolio 3	0.146	-0.014	2.266	0.011
Portfolio HIGH	0.024	11.059	0.024	0.490

Table 4 shows the median values for σ_V observed in the four quartile portfolios, in both Period I and Period II. It appears that these changes are not substantial. We proceed on the assumption that our results are not vulnerable to confounding changes in σ_V.

A similar reasoning could apply for changes in T of the firm. However, since the duration of the debt of a firm is not observed, we are unable to measure the extent to which our experimental design is contaminated by changes in T.

5.4 Change in beta across $\Delta\frac{V}{B}$ quartiles

Table 5 shows the change in beta for the quartile portfolios.

In portfolio LOW, the change in leverage and the change in interest rates are both impacting upon beta in the positive direction. This is borne out by the evidence that the mean beta rose by 0.253, which is significant at a 98.6% level of significance. The change in beta seems to decrease monotonically with Portfolio 2, 3 and HIGH. However, the increase in beta is statistically significant for Portfolio 3 but not for Portfolio 2. In Portfolio HIGH, we expect the worst confounding effects, with a substantial reduction in leverage operating in the opposite direction as compared with the decrease in interest rates. Hence, for Portfolio HIGH, the change in beta seems to be close to 0.

6 Conclusion

In 1996-1998, a remarkable natural experiment took place in India, in the form of monetary policy shocks which generated large changes in interest rates. This gives us a unique opportunity to test the usefulness of the option-pricing approach for thinking about stock betas. In our knowledge, this is the first direct test in the literature on the predicted relationship between interest rates and beta.

We find that the predictions of the theory are substantially validated. Portfolio LOW, where leverage increased and interest rates fell, had a strong increase in betas. Portfolio HIGH, where leverage substantially decreased giving a countervailing effect, had betas stay roughly unchanged. The intermediate portfolios exhibit intermediate results, with an anomaly in the form of a statistically significant change in beta for Portfolio 3 but not 2.

These results constitute some empirical evidence of the usefulness of the option-pricing approach to corporate finance. The model has many simplifications, such as an assumption that all debt is due at $T = 1$. Yet, the predictions of the model in terms of the impact of interest rates upon beta seem to be broadly successful.

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A Deriving Equation 5

Applying Ito's lemma on Equation 2, the returns on the equity can be written as:

$$\Delta E = E_V \Delta V + \frac{1}{2} S_{VV} \sigma_V^2 V^2 \Delta t + E_t \Delta t$$

which gives the stock returns r_E as:

$$\lim_{t \to 0} \bar{r}_E \equiv \frac{E_V}{E} V \frac{\Delta V}{V} \equiv \frac{E_V}{E} V \bar{r}_v$$

Using the stock returns and the market returns $r_M = \frac{\Delta M}{M}$, the beta of the stock can be written as follows:

$$\beta_E \equiv \frac{cov(\bar{r}_E, \bar{r}_M)}{\sigma^2_{\bar{r}_M}} = \frac{E_V}{E} V \frac{cov(\bar{r}_V \bar{r}_M)}{\sigma^2_{\bar{r}_M}} = \frac{E_V}{E} V \beta_V$$

Using the derivative of Equation 2 with respect to V we get:

$$\beta_E = \mathrm{N}(\mathrm{d}_1) \frac{\mathrm{V}}{\mathrm{E}} \beta_{\mathrm{V}} \tag{10}$$

It proves to be useful to think of this as $\beta_E = \eta_E \beta_V$ where $\eta_E = N(d_1) \frac{V}{E}$ is the elasticity of the stock price to changes in the value of the firm.

B Partial Derivative of β_E w.r.t. r_F

$$Q \equiv \frac{VN(d_1) B e_f^{-R} T N(d2)}{E^2 \sigma_v \sqrt{T}} > 0$$

Taking the partial derivative of β_E w.r.t. r_F we get

$$\frac{\partial \beta_E}{\partial r_F} = -QT \left[\frac{Z(d_1)}{N(d_1)} - \frac{Z(d_2)}{N(d_2)} + \sigma_v \sqrt{T} \right] \beta_v \tag{11}$$

It can be shown[3] that that

$$h(d) = \frac{Z(d)}{N(d)} + d > 0 \qquad \forall \qquad -\infty < d < \infty$$

It can be shown that $h'(d) \geq 0$ for all d. As $d_1 > d_2$ therefore

$$h(d_1) - h(d_2) = \frac{Z(d_1)}{N(d_1)} - \frac{Z(d_2)}{N(d_2)} + \sigma_v \sqrt{(T)} > 0.$$

This proves that $\frac{\partial \beta_E}{\partial r_F} < 0$.

[3]Using the upper bound of the Mills ratio(Gordon 1941).