

# **Nonlinear Adjustment in Real Exchange Rates and Long Run**

## **Purchasing Power Parity – Further Evidence**

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### Abstract

The paper investigates, using a threshold autoregression model, the nature of nonlinear adjustments in real exchange rates (RERs) arising from the presence of transaction costs and uncertainty, and their implications for the testing of unit roots. Using monthly data for the U.S. vis-à-vis 19 trading partners we find that most RERs are better characterized by a mean reverting nonlinear stochastic process, with large changes converging faster than small changes, implying that there is convergence towards PPP equilibrium at least in the long-run, albeit in a non-linear manner. It is found that, across countries and commodity groups, there is an association between geographical and trade related proximity and the estimated speeds of adjustment. In addition, policy agreements that mitigate exchange rate uncertainty such as the Louvre Accord could have contributed to greater international commodity arbitrage.

JEL classification: F31, F40, L16, C32

Key words: exchange rates, PPP, threshold autoregression models, unit roots

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## 1. Introduction

The doctrine of purchasing power parity (PPP), or its variant the Law of One Price (LOOP), in its absolute form states that a common basket of goods, expressed in terms of a single currency, costs the same in all countries<sup>1</sup>. The parity condition rests on the assumption of perfect inter-country commodity arbitrage, and is the central building block of many theoretical and empirical models of exchange rate determination (see Rogoff (1996) and Froot and Rogoff (1995) for recent reviews of this literature). Due to factors like transaction costs, taxation, subsidies, trade restrictions and other non-tariff barriers, foreign exchange market interventions, imperfect competition, existence of non-traded goods, and the differential quality and composition of market baskets of commodities and price indices across countries, one may expect PPP to hold only in the long-run and not in the short-run.

For the most part, the empirical literature has found that real exchange rates (RERs) tend toward PPP in the very long run (i.e., it takes many years rather than a few months). The speed of convergence to PPP is extremely slow; deviations appear to damp out at a rate of roughly 15 percent per year. In fact a large body of work (Adler and Lehman (1983) and Huizinga (1987), to name a few) found that RERs follow a random walk. Using standard unit root tests, Corbae and Ouliaris (1988), Meese and Rogoff (1988) and Grilli and Kaminsky (1991) cannot reject the null of unit root for RERs for the managed float regime, with the implication that deviations from PPP persist over long periods of time. In contrast, Frankel and Rose (1996), Lothian (1997), Pappel (1997) and Pappel and Theodoridis (1998) find strong evidence of mean reversion in RERs by

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<sup>1</sup> In its relative version, PPP states that the rate of change in the nominal exchange rate equals the differential between the growth rates in home and foreign price indices.

implementing the panel data variants of standard unit root tests. O'Connell (1998), on the other hand, finds that once the cross-sectional dependence in the exchange rates is accounted for, one cannot reject the unit root hypothesis even in panel data.

Recently, a series of alternative explanations base the persistence of deviations from PPP on the presence of market frictions that impede inter-country commodity trade and arbitrage<sup>2</sup>. Dumas (1992), Uppal (1993), Sercu, Uppal and Van Hulle (1995) and Coleman (1995) develop dynamic general equilibrium models of real exchange rate determination which take into account transaction costs and show that the process of adjustment of RERs towards a PPP equilibrium is non-linear. These models predict that there exist some "bands of inaction" in exchange rate adjustment whose width is related to the uncertainty regarding the permanence of the shocks causing price changes, within which arbitrage is not profitable due to sunk costs (Dixit (1989) and Krugman (1989)). These costs include, *inter alia*, transport costs, trade barriers and costs of setting up or buying foreign retail distribution networks. As a result, deviations from PPP within these bands are left uncorrected. However, deviations outside these bands (i.e., where price differences exceed transaction costs) will be arbitrated away by market forces. Similarly, adjustments in relative prices across borders will also be affected by the perceived uncertainty in exchange rate movements. For example, with a greater degree of uncertainty, firms become less willing to change their prices since the exchange rate may move back after the price change and another price change in the opposite direction may then be necessary (Delgado (1991)). Empirically, implications of some of the above arguments are that deviations from PPP follow a nonlinear stochastic process that is

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<sup>2</sup> Long ago, Heckscher (1916) argued that international transaction costs could prevent deviations from PPP correcting themselves out in the short-run.

mean-reverting with the degree of mean reversion differing across different thresholds, and that the degree of mean reversion in RERs is related to factors such as geographical and trade related proximity across countries, market structure and uncertainty about exchange rate movements.

A series of papers has emerged that attempt to test these predictions<sup>3</sup>. Using exponential smooth transition autoregression (ESTAR) models that allow for the degree of mean reversion to differ across regimes that change smoothly, Michael, Nobay and Peel (1997) and Baum, Cagalayan and Barkoulas (1998) find that the mean reversion is significant for sizeable deviations from PPP<sup>4</sup>. Obstfeld and Taylor (1997) use a threshold autoregression (TAR) model that allows for convergence speeds to differ across two separate regimes - one when deviation from PPP is inside the band and the other when it is outside the band - and find that the degree of mean reversion is stronger in the latter than in the former. In addition, Cheung, Chinn and Fujii (1999) find that exchange rate uncertainty has a negative effect on mean reversion coefficients.

An important limitation of most of the above studies is that they assume that the RERs are (trend) stationary and detrend the series under consideration prior to fitting a TAR model<sup>5</sup>. Such an approach has serious implications for the statistical tests as well as the estimates of convergence speeds. First, by assuming that RERs are (trend) stationary,

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<sup>3</sup> Pippenger and Goering (1993) find that the standard ADF tests for unit roots have very low power against non-linear stationary alternatives, implying that even if the true data generating process for the real exchange rate is a threshold autoregression type non-linear stationary process, these tests typically fail to reject the null of unit root. The paper underscores that ignoring non-linear adjustments in modeling RERs may have contributed to so many other studies finding unit roots.

<sup>4</sup> Michael, Nobay and Peel (1996) analyze historical (annual) data over two hundred years, whereas Baum, Cagalayan and Barkoulas (1999) analyze multi-country (monthly) data on RERs from the post-Bretton Woods period.

<sup>5</sup> This procedure is justified by arguing that these studies were not concerned with modeling the long-run trend behavior of the equilibrium price difference, but only with the short-run properties of the adjustment towards equilibrium.

these papers do not attempt to reconcile the non-linear adjustment they find with the evidence in favor of unit roots that past studies have found. Second, if the assumption of (trend) stationarity is not valid and the RERs contain unit roots in all regimes, their tests of non-linearity will lead to incorrect inferences, as these tests will have non-standard asymptotic distributions. Third, even if the assumption of trend stationarity in RERs is valid, a linear detrending of these series prior to fitting a TAR model can seriously bias the estimates of the convergence speeds if the drift and trend coefficients in RERs differ across thresholds.

As for the finding that estimated mean reversion coefficients are negatively related to the uncertainty in exchange rate movements, it is difficult to give a causal interpretation because of the potential reverse feedback between exchange rate volatility and the speed of adjustment in relative prices. For example, not only does exchange rate volatility make potential arbitrageurs less responsive to exchange rate changes, the low responsiveness itself may raise the amplitude of equilibrium RER fluctuations (Krugman (1989)).

This paper attempts to overcome these limitations by the explicit and simultaneous modelling of possible non-stationarity and TAR type non-linearity in RERs using the threshold autoregression-unit root (TAR-UR) test procedures of Caner and Hansen (1998). To the extent these tests allow for the joint consideration of non-linearity (thresholds) and non-stationarity (unit roots), the present work does not suffer from the limitations mentioned earlier. In addition, the framework allows us to estimate (conditional) convergence speeds of large versus small deviations from PPP without imposing any assumption on long-run trending behaviour and hence provides more

accurate estimates than those of previous studies. To control for the potential endogeneity of variations in RERs, we propose to examine the speeds of adjustment in RERs over different sub-periods associated with changes in the international exchange rate regime, such as the Plaza and Louvre Accords, which have implications for exchange rate uncertainty.

The rest of the paper is organised as follows. Section 2 outlines the general form of the TAR model that is estimated, including the proposed tests for non-linearity and unit roots. In Section 3, these tests are applied to a sample of 19 bilateral RERs pertaining to the U.S vis-à-vis its trading partners. We also investigate the relationship between the speed of convergence in RERs and geographical and trade related proximity, and the impact of changes in exchange rate arrangements among the major industrialized countries on international commodity arbitrage. We conclude in Section 4 and provide directions for future research.

## 2. Econometric Methodology

Following recent developments in the non-linear time series literature, we attempt to test for unit roots in a TAR model of real exchange rates. The proposed empirical model is the following threshold autoregression<sup>6</sup>:

$$\Delta y_t = \mathbf{q}'_1 x_{t-1} \mathbf{I}_{\{z_{t-1} < l\}} + \mathbf{q}'_2 x_{t-1} \mathbf{I}_{\{z_{t-1} \geq l\}} + e_t, \quad (1)$$

$t = 1, \dots, T$ , where  $x_{t-1} = (y_{t-1} \ t-1 \ \Delta y_{t-1} \ \dots \ \Delta y_{t-k})'$ ,  $\mathbf{I}_{\{.\}}$  is an indicator function that takes a value of 1 when the argument inside  $\{.\}$  holds and 0 otherwise,  $e_t$  is an iid error and

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<sup>6</sup> The material in this section is heavily borrowed from Caner and Hansen (1998).

$Z_t = y_t - y_{t-m}$  for some  $m \geq 1$ . The particular specification for the threshold variable  $Z_{t-1}$  is not essential to the analysis. In general, what is necessary is that  $Z_{t-1}$  is predetermined, strictly stationary and ergodic with a continuous distribution function. The threshold  $\lambda$  is unknown; it takes on values in the interval  $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ , where  $\lambda_1$  and  $\lambda_2$  are picked so that  $P(Z_t \leq \lambda_1) = \pi_1 > 0$  and  $P(Z_t \leq \lambda_2) = \pi_2 < 1$ . The specification of  $\pi_1$  and  $\pi_2$  is inherently arbitrary, and in practice must be guided by the consideration that each "regime" needs to have sufficient observations to adequately identify the regression parameters<sup>7</sup>.

For some of our analysis, it is convenient to explicitly partition vectors  $\theta_1$  and  $\theta_2$  as follows:

$$\mathbf{q}_1 = \begin{pmatrix} \mathbf{r}_1 \\ \mathbf{b}_1 \\ \mathbf{m}_1 \\ \mathbf{a}_1 \end{pmatrix} \quad \text{and} \quad \mathbf{q}_2 = \begin{pmatrix} \mathbf{r}_2 \\ \mathbf{b}_2 \\ \mathbf{m}_2 \\ \mathbf{a}_2 \end{pmatrix}$$

where  $\alpha_1$  and  $\alpha_2$  are each  $k$ -vectors containing the slope coefficients on  $k$ -lagged dependent variables  $(\Delta y_{t-1}, \dots, \Delta y_{t-k})$  in the two regimes. The remaining parameters are scalar:  $(\mu_1, \mu_2)$  are the intercepts,  $(\beta_1, \beta_2)$  are the trend slopes and  $(\rho_1, \rho_2)$  are the slope coefficients on  $y_{t-1}$ .

For each  $\lambda \in \Lambda$ , the TAR model (1) is estimated by ordinary least squares (OLS):

$$\Delta y_t = \hat{\mathbf{q}}_1(\mathbf{I})' x_{t-1} \mathbf{I}_{\{Z_{t-1} < \lambda\}} + \hat{\mathbf{q}}_2(\mathbf{I})' x_{t-1} \mathbf{I}_{\{Z_{t-1} \geq \lambda\}} + \hat{e}_t. \quad (2)$$

Let,

$$\hat{\mathbf{S}}^2(\mathbf{I}) = T^{-1} \sum_1^T \hat{e}_t(\mathbf{I})^2$$

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<sup>7</sup> For the empirical work reported in the next section we set  $\pi_1 = 0.15$  and  $\pi_2 = 0.85$ . These choices impose the restriction that neither "regime" has less than 15 percent of the total sample.

be the OLS estimate of  $\sigma^2$  for fixed  $\lambda$ . The least-squares (LS) estimate of the threshold  $\lambda$  is found by minimizing  $\sigma^2(\lambda)$ :

$$\hat{\mathbf{I}} = \arg \min_{\mathbf{I} \in \Lambda} \hat{\mathbf{S}}^2(\mathbf{I}).$$

The LS estimates of the other parameters are then found by plugging in the point estimate

$\hat{\mathbf{I}}$ , vis.,

$$\hat{\mathbf{q}}_1 = \hat{\mathbf{q}}_1(\hat{\mathbf{I}}) \quad \text{and} \quad \hat{\mathbf{q}}_2 = \hat{\mathbf{q}}_2(\hat{\mathbf{I}}).$$

We can therefore write the estimated model as

$$\Delta y_t = \hat{\mathbf{q}}_1'(\hat{\mathbf{I}})' x_{t-1} \mathbf{I}_{\{z_{t-1} < r\}} + \hat{\mathbf{q}}_2'(\hat{\mathbf{I}})' x_{t-1} \mathbf{I}_{\{z_{t-1} \geq r\}} + \hat{e}_t, \quad (3)$$

which also defines the LS residuals  $e_t$ .

The estimates from (3) can be used for making inferences concerning the parameters of (1) using standard Wald statistics. We are particularly interested in restrictions concerning the presence of a threshold and a unit root. First, the threshold effect disappears under the joint hypothesis

$$H_0: \theta_1 = \theta_2. \quad (4)$$

Our test of (4) is the standard Wald statistic  $W_T$  for this restriction. To establish notation, let

$$W_T(\mathbf{I}) = T \left( \frac{\hat{\mathbf{S}}_0^2}{\hat{\mathbf{S}}^2(\mathbf{I})} - 1 \right)$$

denote the Wald statistic for hypothesis (4) for fixed  $\lambda$  from regression (2), where  $\sigma_0^2$

is the residual variance from the OLS estimation of the null linear model. It is useful to note that since  $W_T(\lambda)$  is a decreasing function of  $\sigma^2(\lambda)$ , the following relationship is obtained:

$$W_T = W_T(\hat{\mathbf{I}}) = \sup_{\mathbf{I} \in \Lambda} W_T(\mathbf{I}).$$

The other hypothesis of major interest is the presence of a unit root in the autoregressive structure. A unit root in  $y_{t-1}$  occurs in (1) when

$$H_0: \rho_1 = \rho_2 = 0. \quad (5)$$

The standard test for (5) is the Wald statistic  $R_T$  from (3). To fix notation, let  $R_T(\lambda)$  be the standard Wald statistic for hypothesis (5) for fixed  $\lambda$ :

$$R_T = R_T(\hat{\mathbf{I}}).$$

The statistic  $R_T$  may be viewed as a two-parameter generalization of the standard Dickey-Fuller statistic.

In sum, from the estimates obtained from (3) we have proposed two Wald tests -  $W_T$  and  $R_T$  - which test restrictions on the coefficients implying, respectively, the absence of threshold effects and the presence of a unit root. While the statistics are standard, their sampling distributions are non-standard. The Wald test for threshold effects has a non-standard asymptotic null distribution due to the presence of a parameter  $\lambda$  (which is not identified under the null hypothesis) and, partially, due to the assumption of near non-stationary autoregression. The Wald test for a unit root on the other hand, has an asymptotic null distribution, which depends upon whether or not there is a true threshold effect, but it is free of nuisance parameters. Following Caner and Hansen (1998), we compute the p-values associated with the test statistics from bootstrap simulations.

### **3. Data set and Results**

#### **3.1 Data**

The main variable of interest is the real exchange rate computed as the relative price ratio of a basket of commodities in two countries expressed in a common currency. Commodity Price Index (CPI) for both aggregate and disaggregate commodity groups is used for measuring price levels in each country. In all cases, the US is considered as the home country and the nominal exchange rates that are used are end-of-month bilateral US dollar exchange rates. The study is undertaken for the period 1978-1998 for a broad set of U.S. trading partners: 19 countries for aggregate CPI based measures, and 8 to 13 countries for disaggregated commodity group consumer price indices.

All series, except the disaggregated commodity-wise price indices, are from the International Monetary Fund's *International Financial Statistics Database*. The countries analyzed with aggregate CPI are: Austria, Belgium, Canada, Denmark, France, Germany, Greece, Italy, Japan, Korea, Mexico, Netherlands, Norway, Portugal, Singapore, Spain, Sweden, Switzerland and the UK. The disaggregated commodity-group wise data - for food and transportation - is the Engel and Roger data set used in Engel, Hendrickson and Rogers (1997). The countries in this data set include Austria, Belgium, Denmark, France, Germany, Greece, Hong Kong, Japan, Korea, Mexico, Netherlands, Norway, Singapore, Spain and the UK.

#### **3.2 Results**

Before proceeding with the discussion of the results of the TAR model, two comments are in order. First, unlike previous studies, the threshold variable in our study is the cumulative lagged change in the dependent variable (the long-difference). In fact,

we take the absolute of the lagged value of long-difference of a series, so that the estimated model is a double threshold AR model, with one type of adjustment within the band and another type of adjustment outside the band. In addition, we do not fix a priori the delay parameter,  $m$ , that determines the cumulated change in real exchange rates from  $m$  periods lagged. Instead, we endogenously determine it by choosing an optimal  $m$  that minimizes the error sum of squares following an iterative procedure that is similar to that used for estimating  $\lambda$ <sup>8</sup>. Second, since we allow higher order dynamics in equation (1), the choice of lag length becomes an important issue. Unlike in the linear autoregression models, the standard Akaike Information Criterion (AIC) is found to be quite misleading in detecting the appropriate lag length in TAR models. Following Wong and Li (1998), we use a bias corrected AIC (AICc) to choose the lag length,  $p$ , for the lagged dependent variable in estimating equation (1). It should be noted, however, that the power properties of AICc in the context of testing for unit roots in TAR models are not well understood. As mentioned above, the TAR model is estimated for aggregate price indices (CPI-overall) as well as two disaggregated commodity price indices - CPI-food commodities and CPI-transport services - for a number of countries, for the period 1978 to 1998. The choice of the latter two series is motivated by the fact that they broadly represent, respectively, tradable and non-tradable commodity groups.

To link our work with previous studies in the literature that test for a unit root in real exchange rate series, we carry out a detailed univariate analysis of the long-run trends in the latter using the standard tests for unit roots. The results of the ADF tests reported in Table 1 show that for all series under consideration we cannot reject the null of unit root in RERs, as well as relative price ratios at 5 percent level of significance.

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<sup>8</sup> We also estimate our model by fixing the delay parameter,  $m$ , to one. The results are qualitatively similar.

Therefore, assuming linear form of adjustments in RERs, as these standard tests do, we cannot reject the hypothesis that deviations from PPP will persist for a long time. The estimated speeds of convergence ( $\rho$ s in equation (3)) and the associated conditional half-lives (CHL) reported in Table 2, also indicate that most of the series have very slow convergence rates and long half-lives, a finding consistent with previous studies.

Next, we turn to the estimates of the TAR-UR model (3). The computed p-values associated with the Wald test for non-linearity reported in Table 3 clearly reject the null hypothesis of a linear AR model in favor of a TAR model, indicating the significance of non-linear adjustments for all exchange rates and relative price ratios under consideration. Although the finding of nonlinear adjustments in RERs is similar to that of some of the recent empirical studies (Obstfeld and Taylor (1997) and Michael, Nobay and Peel (1997)), it is important to note that our finding is robust to the specification of long-run trends in the RERs as we do not impose the assumption of trend stationarity on the data. Having found strong evidence in favor of TAR-type non-linearity, we now turn to test for unit roots in RERs with a TAR model specification. The results for CPI-overall, reported in the second column of Table 4 show that for 14 out of 19 countries, the null of unit root is rejected at 10 percent level of significance. This is an important finding as it implies that once allowance for some degree of non-linear adjustment is made in the model specification, the null of unit root in RERs is strongly rejected. The finding, coupled with the simulation study by Pippenger and Goering (1993), provides a probable explanation for why many previous studies have found the presence of unit roots using linear models. The evidence against unit roots, similarly, is stronger in the case of CPI-food, where the null is rejected for 8 out of 13 series compared to CPI-

transport (an obvious non-tradable), where the null is rejected for only 3 series. The bootstrap p-values reported in Table 4 also indicate that these findings are relatively robust to small sample biases. For all the three RER series reported in Tables 5, 6 and 7, the estimates of  $\rho$  coefficients outside the band (i.e., regime 2) are negative and statistically significant. Hence, these countries' RERs are better characterized by a TAR-type non-linear, but (globally) stationary stochastic process. This would imply that there is convergence towards PPP equilibrium, at least in the long run, albeit in a non-linear manner.

In order to understand the dynamics of adjustment in RERs and relative price ratios better, we analyze the estimated convergence speeds and half-lives of each series across thresholds<sup>9</sup>. The estimates for CPI-overall reported in Table 5 show that the (absolute value of) adjustment coefficients and convergence speeds are higher outside the threshold (regime 2) than within the threshold (regime 1). Similarly, the results also show that the typical half-life of price differences outside the threshold band is 6 to 8 months. In contrast, the linear AR model implies a typical half-life of about 18 to 20 months (see Table 2). We obtain broadly similar results for the relative prices of food commodities, although, not surprisingly, some estimates for CPI-transport do not reflect this pattern (see Tables 6 and 7). We find that our estimates of convergence speeds (half-lives) are typically higher (lower) than those reported in earlier work. This may well be due to the general TAR model specification that we employ which, unlike Obstfeld and Taylor (1997), does not impose any restriction on the nature of long-run trends, the drift coefficients or autoregressive dynamics. Our estimates suggest that the implied threshold

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<sup>9</sup> The adjustment coefficients are allowed to differ from zero under both regimes.

value indicating the "bands of inaction" constitutes about 2 to 9 percent of RERs across various countries and commodity groups.

The results also indicate that while there is a relation between proximity (geographical and otherwise) to the U.S and the threshold value, i.e., countries that are geographically close to the US and / or have greater degree of trade orientation with it have smaller threshold values compared to the others, no such relation is discernible in the convergence speeds. In fact, for CPI-overall, the estimated speeds of convergence (and their ranks reported in the second column of Table 8) point out an apparently counterintuitive pattern in the data, specifically that the speed of adjustment in the RER between the US and Canada is the lowest among all countries. Similarly, for CPI-food the speeds of adjustment for Austria, France and the Netherlands are much higher than that for Japan, Singapore and the UK (second column of Table 9). Considering the possibility that this pattern may be due to differences in the observed variability of real exchange rates and the estimated bandwidth ( $\hat{\lambda}$ ), we compute the ranks of the estimated speeds of convergence for each country normalizing with its coefficient of variation in RER and  $\lambda$ . The results, reported in the last columns of Tables 8 and 9 for CPI-all and CPI-food respectively, clearly indicate that once the degree of variation in RERs and the size of the bandwidth are controlled for, a clear relation emerges between the speeds of adjustment and geographical and other trade related proximity. For example, the speed of adjustment towards PPP equilibrium, for a given degree of variation in RERs and size of the bandwidth indicating say, transaction costs turns out to be much higher for Canada, Singapore, Japan, Korea and the UK than for the other countries. It is interesting to note that on average the Asian countries have much faster adjustment speeds (normalized for

variability) than the European countries. This may be due to increasing trade orientation of the US towards these economies during the floating exchange rate period and because of a high proportion of trade as a percentage of GNP in these economies.

As alluded earlier, adjustments in relative prices across borders will also be affected by the perceived uncertainty in exchange rate movements. For example, with a greater degree of uncertainty, firms become less willing to change their prices since the exchange rate may move back after a price change, and a subsequent price change in the opposite direction may be necessary, thereby underscoring that an increase in exchange rate uncertainty intensifies price stickiness (Delgado (1991)). In terms of our empirical framework this could imply a positive relation between exchange rate uncertainty and the persistence of deviations from PPP equilibrium. To test whether such a relation indeed exists, we need to deploy a measure of exchange rate uncertainty; studies in the past have used standard deviation in nominal exchange rates as one such measure (see Cheung, Chinn and Fujii (1999) for a recent example). These studies, however, do not control for the potential reverse feedback between exchange rate volatility and the speed of adjustment in relative prices. For example, not only does exchange rate volatility make potential arbitrageurs less responsive to exchange rate changes, the low responsiveness itself may raise the amplitude of equilibrium real exchange rate fluctuations (Krugman (1989)).

To control for the potential endogeneity of variation in RERs, we take a slightly different approach by using some historical information about changes in exchange rate regimes that could have had implications for exchange rate uncertainty. Two significant events in international exchange rate arrangements in the floating era are related to the

Plaza and Louvre Accords,<sup>10</sup> whereby agreements were reached among the G7 countries that favored coordinated intervention in foreign exchange markets. In particular, these accords installed a loose regime of reference ranges - within which currencies are to be maintained – to try to impart stability in the foreign exchange markets. To facilitate this, the G-7 established a set of economic indicators to try to guide the effort to coordinate macroeconomic policies with the objective of limiting international imbalances and promoting global growth. By highlighting the commitment of member countries for a coordinated approach to tackle their balance of payments problems, the arrangements have been expected to reduce uncertainty about exchange rate movements in major currencies compared to each country intervening unilaterally (Funabashi (1988)).

To understand the impact of such a change in exchange rate arrangements on international commodity arbitrage, we test whether the estimated speeds of convergence in RERs are different across the pre-1987 and post-1987 eras by adding an interaction dummy variable with a lagged RER variable in equation (3). The estimated speeds of convergence and the associated half-lives outside the band are reported in Table 10 for CPI-all and Table 11 for CPI-food. The results indicate that for both commodity groups in almost all countries, and most notably for the G-7 countries, the speeds of convergence outside the bands are higher during the post-1987 period compared to the pre-1987 period. Similarly, for CPI-all, the half-lives for the pre-1987 period range between 6 to 8 months whereas the typical half-life for the post-1987 period is in the range of 3 to 5 months. For CPI-food the half-lives range, respectively, from 3 to 6 months and 2 to 4 months during these two periods. If one could conclude that coordinated interventions

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<sup>10</sup> The Plaza and Louvre Accords came into effect from September 1985 and February 1987, respectively.

have indeed reduced uncertainty in foreign exchange markets, then our results would imply that such a reduction in uncertainty has resulted in faster international commodity arbitrage<sup>11</sup>. This would complement the evidence in previous studies that exchange rate volatility has an adverse effect on the volume of international trade flows (see McKenzie (1999) for a recent survey).

#### **4. Conclusion**

This paper is an attempt to highlight the importance of the presence of TAR-type non-linear adjustments in real exchange rates arising from impediments to free international arbitrage in commodity trading which can be attributed to, *inter alia*, the existence of sunk costs and uncertainty. We simultaneously and explicitly model TAR-type non-linearities and unit root type non-stationarity in RERs (and relative price differences).

Using a sample of monthly RERs for the period 1978-1998 computed for a broad set of U.S. trading partners and across commodity groups, we find that, for the most part, RERs are better characterised by a TAR-type non-linear stochastic process that is strongly mean reverting as against a linear unit root process; and the degree of mean reversion in deviations from PPP is significantly different across thresholds, with deviations outside the bands converging faster than those within the bands, implying that there is convergence towards PPP equilibrium, at least in the long-run, albeit in a non-linear manner. It would seem that unless there is a strong theoretical reason to believe that the data under consideration is driven by a (log) linear process (such as optimal consumption paths under rational expectations - permanent income hypothesis or asset

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<sup>11</sup> Lopez (1996) finds evidence in favor of cointegration among G7 bilateral exchange rates during the post-Louvre Accord period.

prices following a random walk under efficient markets hypothesis), any inference about unit roots in the data generation process using linear models may be an outcome of neglected non-linearity. This point is similar in spirit to that made by Perron (1989) about how structural breaks in the data can, when neglected, bias inference about unit roots. It would be instructive to analyze the sensitivity of the threshold autoregression model for neglected structural breaks, and we leave this for future research.

Our results reinforce the insight of previous studies regarding the presence of non-linear, but stationary adjustments in RERs, although the estimated convergence speeds in our study are much higher. It is found that there is an association between geographical and trade related proximity and the estimated speeds of adjustment across countries and commodity groups. Evidence put forward in the paper would seem to indicate that policy agreements that mitigate exchange rate uncertainty such as the Louvre Accord could have facilitated greater international commodity arbitrage. This is congruent with findings in many studies that exchange rate volatility has an adverse effect on the volume of international trade flows.

## References

- Adler, M. and Lehman, B. (1983), "Deviations from purchasing power parity in the long run", *Journal of Finance*, 38 (5), 147-87.
- Baum, C., Cagalayan, M. and Barkoulas, J. (1998), "Nonlinear adjustments to purchasing power parity in the post-Bretton Woods era", Working Paper, Boston College, USA.
- Caner, M. and Hansen, B. (1998), "Threshold autoregression with a near unit root", unpublished manuscript, University of Wisconsin, Maddison.
- Cheung, Y., Chinn, M.D. and Fujii, E. (1999), "Market structure and the persistence of sectoral real exchange rates", Working Paper No. 7408, National Bureau of Economic Research, Cambridge, MA.
- Coleman, A.M.G. (1995), "Arbitrage, storage, and the law of one price: New theory for the time series analysis of an old problem", mimeo, Princeton University, USA.
- Corbae, D. and Ouliaris, S. (1988), "Cointegration based tests of purchasing power parity", *Review of Economics and Statistics*, 70, 508-511.
- Delgado, F.A (1991), "Hysteresis, menu costs and pricing with random exchange rates", *Journal of Monetary Economics*, 28 (3), 461-484.
- Dixit, A. (1989), "Hysteresis, import penetration, and exchange rate pass-through", *Quarterly Journal of Economics*, 104, 205-228.
- Dumas, B. (1992), "Dynamic equilibrium and real exchange rates in a spatially separated world", *Review of Financial Studies*, 5 (2), 153-180.
- Engel, C., Hendrickson, M. and Rogers, J. (1997), "Intra-national, intra-continental and intra-planetary PPP", NBER Working Paper No: 6069.
- Frankel, J.A. and Rose, A. (1996), "Mean reversion within and between countries: A panel project on purchasing power parity", *Journal of International Economics*, 40, 209-224.
- Froot, K. and Rogoff, K. (1995), "Perspectives on PPP and long-run real exchange rates", Chapter 32 in *Handbook of International Economics*, G. Grossmann and K. Rogoff eds. Amsterdam, North-Holland.
- Funabashi, Y. (1988), "Managing the dollar: From the Plaza to the Louvre", Washington, D.C.: Institute of International Economics.
- Grilli, V. and Kaminsky, G. (1991), "Nominal exchange rate regimes and the real exchange rate", *Journal of Monetary Economics*, 27, 191-212.

Heckscher, E.F. (1916), "Växelkursens grundval vid pappersmyntfot", *Ekonomisk Tidskrift* 18 (October), 309-312.

Huizinga, J. (1987), "An empirical investigation of the long-run behavior of real exchange rates", *Carnegie-Rochester Conference Series on Public Policy*, 27, 149-214.

Krugman, P. R. (1989), "Exchange rate instability", Cambridge, MA: MIT press.

Lopez, J.A. (1996), "Exchange rate cointegration across central bank regime shifts", *Federal Reserve Bank of New York Research Paper No: 9602*, Federal Reserve Bank of New York, NY.

Lothian, J. (1997), "Multi-country evidence on the behavior of purchasing power parity under the current float", *Journal of International Money and Finance*, 16, 19-35.

McKenzie, M.D. (1999), "The impact of exchange rate volatility on international trade flows", *Journal of Economic Surveys*, 13, 71-106.

Meese, R.A. and Rogoff, K.S. (1988), "Was it real? The exchange rate interest differential relation over the modern floating-rate period", *Journal of Finance*, 43 (4), 933-948.

Michael, P., Nobay, A.R. and Peel, D.A. (1997), "Transaction costs and nonlinear adjustment in real exchange rates: An empirical investigation", *Journal of Political Economy*, 105 (4), 862-79.

Obstfeld, M. and Taylor, A.M. (1997), "Nonlinear aspects of goods-market arbitrage and adjustment: Heckscher's commodity points revisited", *Journal of the Japanese and International Economies*, 11, 441-479.

O'Connell, P.G. (1998), "Market frictions and real exchange rates", *Journal of International Money and Finance*, 17, 71-95.

Pappel, D. (1997), "Searching for stationarity: Purchasing power parity under the current float", *Journal of International Economics*, 43, 313-332.

Pappel, D., and Theodoridis, H. (1998), "Increasing evidence of purchasing power parity over the current float", *Journal of International Money and Finance*, 17, 41-50.

Perron, P. (1989), "The great crash, the oil price shock and the unit root hypothesis", *Econometrica*, 57, 1361-1401.

Pippenger, M.K., and Goering, G.E. (1993), "A note on the empirical power of unit root tests under Threshold processes", *Oxford Bulletin of Economics and Statistics*, 55, 472-481.

Rogoff, K. (1996), "The purchasing power parity puzzle", *Journal of Economic Literature*, 34, 647-668.

Sercu, P., Uppal, R., and Van Hulle, C. (1995), "The exchange rate in the presence of transaction costs: implications for tests of purchasing power parity", *Journal of Finance*, 50, 1309-1319.

Uppal, R. (1993), "A general equilibrium model of international portfolio choice", *Journal of Finance*, 48 (2), 529-553.

Wong, C.S. and Li, W.K. (1998), "A note on the corrected Akaike Information Criterion for Threshold Autoregressive models", *Journal of Time Series Analyses*, 19, 113-124.

**Table 1**  
**Linear AR model : ADF statistics**

Country	CPI (all)	CPI (food)	CPI (trnsp.)
Austria	-2.45	-3.05	
Belgium	-2.21	-3.29	
Canada	-1.81		
Denmark	-2.14	-2.99	
France	-1.96	-3.2	
Germany	-2.42	-3.19	
Greece	-2.15	-2.18	-3.32
Hong Kong		-2.08	-3.29
Italy	-2.36		
Japan	-2.4	-2.44	-2.89
Korea	-2.07	-2.02	
Mexico	-2.96		-2.56
Netherlands	-1.89	-3.22	
Norway	-1.99	-2.48	-2.51
Portugal	-2.06		
Singapore	-2.04	-0.83	-1.68
Spain	-2.35		-2.65
Sweden	-2.16		
Switzerland	-2.35		-3.35
UK	-2.35	-2.68	

*Note: Critical value at 5 percent is -3.43*

**Table 2**  
**Estimates of convergence speeds ( $\rho$ ) and conditional half-lives (CHL)**  
**(Linear AR model)**

Country	CPI (all)		CPI (food)		CPI (trnsp.)	
	$\rho$	CHL	$\rho$	CHL	$\rho$	CHL
Austria	-0.032	21.3	-0.037	18.4		
Belgium	-0.025	27.4	-0.04	17.0		
Canada	-0.021	32.7				
Denmark	-0.026	26.3	-0.037	18.4		
France	-0.025	27.8	-0.038	17.9		
Germany	-0.031	22.0	-0.03	22.8		
Greece	-0.028	24.4	-0.02	34.3	-0.03	22.8
Hong Kong			-0.025	27.4	-0.03	22.8
Italy	-0.03	22.8				
Japan	-0.033	20.7	-0.02	34.3	-0.028	24.4
Korea	-0.037	18.4	-0.005	138.3		
Mexico	-0.065	10.3			-0.049	13.8
Netherlands	-0.024	28.5	-0.0275	24.9		
Norway	-0.031	22.0	-0.036	18.9	-0.029	23.6
Portugal	-0.022	31.2				
Singapore	-0.021	32.7	-0.027	25.3	-0.023	29.8
Spain	-0.027	25.3			-0.044	15.4
Sweden	-0.026	26.3				
Switzerland	-0.037	18.4			-0.076	8.8
UK	-0.039	17.4	-0.0296	23.0		

*Note: CHL denotes the number of months it takes for 50 percent of the shock to die out.*

**Table 3**  
**Tests for linear AR model versus TAR model**

Country	CPI (all)		CPI (food)		CPI (trnsp.)	
	$W_T$ (p-value)	p,m	$W_T$ (p-value)	p,m	$W_T$ (p-value)	p,m
Austria	30.7 (0.028)	8,5	20.4 (0.090)	10,6		
Belgium	52.5 (0.002)	9,6	30.1 (0.028)	10,4		
Canada	26.4 (0.041)	12,3				
Denmark	47.3 (0.005)	11,6	32.2 (0.026)	10,4		
France	29.5 (0.031)	10,6	29.6 (0.033)	9,6		
Germany	41.9 (0,011)	11,4	23.8 (0.078)	10,8		
Greece	41.3(0.014)	11,2	20.4 (0.101)	10,2	21.4 (0.089)	11,2
Hong Kong			22.4 (0.078)	9,5	38.9 (0.022)	10,2
Italy	26.2 (0.044)	5,4				
Japan	23.8 (0.052)	5,5	37.8 (0.019)	10,3	22.9 (0.080)	10,2
Korea	31.5 (0.021)	10,1	30.7 (0.031)	10,8		
Mexico	52.9 (0.001)	12,6			36.6 (0.03)	10,9
Netherlands	29.9 (0.020)	6,6	31.0 (0.029)	10,6		
Norway	51.4 (0.004)	12,1	32.4 (0.027)	9,4	32.9 (0.027)	10,1
Portugal	24.5 (0.050)	12,9				
Singapore	32.0 (0.022)	1,1	28.1 (0.043)	7,1	20.6 (0.09)	3,3
Spain	47.7 (0.007)	11,1			46.9 (0.01)	10,9
Sweden	27.8 (0.037)	11,9				
Switzerland	24.8 (0.050)	10,6			35.9 (0.03)	11,2
UK	29.6 (0.022)	10,4	21.7 (0.086)	10,4		

*Note: p-values are calculated from 300 bootstrap simulations.*

**Table 4**  
**Tests for unit roots in TAR model**

Country	CPI-all (p-value)	CPI-food (p-value)	CPI-trnsp.(p-value)
Austria	17.6 (0.035)	17.2 (0.038)	
Belgium	17.8 (0.034)	23.6 (0.03)	
Canada	9.1 (0.410)		
Denmark	16.3 (0.056)	25.0 (0.001)	
France	17.8 (0.034)	21.7 (0.008)	
Germany	16.8 (0.046)	15.2 (0.050)	
Greece	9.31 (0.400)	13.2 (0.110)	11.4 (0.202)
Hong Kong		24.9 (0.001)	11.1 (0.232)
Italy	22.1 (0.006)		
Japan	19.8 (0.016)	10.3 (0.290)	14.5 (0.059)
Korea	19.9 (0.015)	20.3 (0.012)	
Mexico	18.9 (0.022)		13.4 (0.118)
Netherlands	14.9 (0.051)	28.3 (0.001)	
Norway	11.8 (0.201)	9.04 (0.390)	5.4 (0.732)
Portugal	10.9 (0.270)		
Singapore	17.4 (0.035)	13.1 (0.120)	6.4 (0.623)
Spain	10.3 (0.320)		7.9 (0.524)
Sweden	15.9 (0.048)		
Switzerland	15.1 (0.050)		15.4 (0.050)
UK	15.9 (0.049)	11.6 (0.210)	

*Note: Critical value at 5 percent is 14.39. p-values are generated from 300 bootstrap simulations.*

**Table 5**  
**Estimates of convergence speeds and conditional half-lives for RER - CPI-all**  
**(TAR model)**

Country	Threshold ( $\lambda$ )	$\rho_1$	$\rho_2$	CHL1( $\rho_1$ )	CHL2( $\rho_2$ )	p,m
Austria	0.072	0.004	-0.090	28.5	7.4	8,5
Belgium	0.110	-0.013	-0.140	98.7	4.6	9,6
Canada	0.027	-0.011	-0.035	62.7	19.5	12,3
Denmark	0.098	-0.030	-0.110	230.7	6.0	11,6
France	0.124	-0.017	-0.150	98.7	4.3	10,6
Germany	0.086	-0.024	-0.098	38.2	6.7	11,4
Greece	0.064	-0.002	-0.060	98.7	11.2	11,2
Italy	0.104	-0.017	-0.158	692.8	4.3	5,4
Japan	0.149	-0.024	-0.170	69.0	3.7	5,5
Korea	0.062	-0.033	-0.160	34.3	4.0	10,1
Mexico	0.102	-0.040	-0.270	10.5	2.3	12,6
Netherlands	0.085	-0.030	-0.082	138.3	8.1	6,6
Norway	0.066	-0.029	-0.078	24.4	8.6	12,1
Portugal	0.063	-0.003	-0.060	138.3	11.2	12,9
Singapore	0.029	-0.018	-0.100	69.0	6.6	1,1
Spain	0.063	-0.011	-0.080	25.3	8.3	11,1
Sweden	0.120	-0.019	-0.100	57.4	6.6	11,9
Switzerland	0.126	-0.039	-0.097	26.3	6.8	10,6
UK	0.117	-0.023	-0.113	38.7	5.8	10,4

*Note: The formula for (conditional) half-life is  $\ln(0.5)/\ln(1+\rho)$ .  $\rho_1$  and CHL1 correspond, respectively, to the adjustment speed and half-life in Regime 1 (within the estimated band, i.e., (+threshold, -threshold)), while  $\rho_2$  and CHL2 correspond to those outside the band.*

**Table 6****Estimates of convergence speeds and conditional half-lives for RER - CPI-food**

Country	Threshold ( $\lambda$ )	$\rho_1$	$\rho_2$	CHL1	CHL2	p,m
Austria	0.14	-0.035	-0.17	19.5	3.7	10,6
Belgium	0.07	-0.015	-0.14	45.9	4.6	10,4
Denmark	0.06	-0.027	-0.11	25.3	6.0	10,4
France	0.135	-0.04	-0.23	17.0	2.7	9,6
Germany	0.11	-0.052	-0.11	13.0	6.0	10,8
Greece	0.08	-0.02	-0.09	34.3	7.4	10,2
HongKong	0.04	-0.046	-0.17	14.7	3.7	9,5
Japan	0.045	-0.005	-0.11	138.3	6.0	10,3
Korea	0.14	-0.005	-0.16	138.3	4.0	10,8
Netherlands	0.13	-0.026	-0.23	26.3	2.7	10,6
Norway	0.07	-0.04	-0.1	17.0	6.6	10,4
Singapore	0.02	-0.02	-0.1	34.3	6.6	7,1
UK	0.1	-0.026	-0.15	26.3	4.3	10,1

*See note at bottom of Table 5.*

**Table 7****Estimates of convergence speeds and conditional half-lives for RER - CPI-transport**

Country	Threshold ( $\lambda$ )	$\rho_1$	$\rho_2$	CHL1	CHL2	p,m
Greece	0.07	-0.027	-0.15	25.3	4.3	11,2
HongKong	0.038	-0.036	-0.096	18.9	6.9	10,2
Japan	0.039	-0.039	-0.11	17.4	6.0	10,2
Mexico	0.1	-0.068	-0.11	9.8	6.0	10,9
Norway	0.02	-0.02	-0.06	34.3	11.2	10,1
Singapore	0.01	-0.002	-0.02	346.2	34.3	3,3
Spain	0.16	-0.039	-0.079	17.4	8.4	10,9
Switzerland	0.035	-0.12	-0.04	5.4	17.0	11,2

*See note at bottom of Table 5.*

**Table 8****Ranking of estimated convergence speeds for RER - CPI-all  
(TAR model)**

Country	Rank of $\rho_2$	CV(RER)	Rank of $\rho_2'$ <sup>a</sup>
Austria	13	0.430	6
Belgium	6	1.630	15
Canada	19	0.116	2
Denmark	8	0.590	12
France	5	1.150	14
Germany	11	8.070	18
Greece	18	7.970	19
Italy	2	0.760	11
Japan	3	0.210	3
Korea	4	0.680	4
Mexico	1	1.190	9
Netherlands	14	1.690	16
Norway	15	4.040	17
Portugal	17	0.450	10
Singapore	9	0.230	1
Spain	16	0.570	8
Sweden	10	0.470	13
Switzerland	12	0.340	7
UK	7	0.320	5

<sup>a</sup> $\rho_2' = \rho_2 / (CV(RER) * \lambda)$ .

**Table 9****Ranking of estimated convergence speeds for RER - CPI-food  
(TAR model)**

Country	Rank of $\rho_2$	CV(RER)	Rank of $\rho_2'$ <sup>a</sup>
Austria	3	0.174	11
Belgium	7	0.167	6
Denmark	8	0.134	5
France	1	0.192	9
Germany	9	0.095	7
Greece	13	0.960	13
HongKong	4	0.108	2
Japan	10	0.101	4
Korea	5	0.120	8
Netherlands	2	0.220	10
Norway	11	0.530	12
Singapore	12	0.057	1
UK	6	0.060	3

<sup>a</sup> $\rho_2' = \rho_2 / (CV(RER)*\lambda)$ .

**Table 10**  
**Estimates of convergence speeds and conditional half-lives for RER - CPI-all**  
**(Pre-and Post-1987 (TAR model))**

Country	Threshold ( $\lambda$ )	$\rho_2$		CHL( $\rho_2$ )		p,m
		Pre - 1987.3	Post - 1987.3	Pre - 1987.3	Post - 1987.3	
Austria	0.072	-0.110	-0.119	5.9	5.5	8,5
Belgium	0.110	-0.146	-0.182	4.4	3.5	9,6
Canada	0.027	-0.022	-0.059	31.2	11.4	12,3
Denmark	0.098	-0.123	-0.153	5.3	4.2	11,6
France	0.124	-0.145	-0.176	4.4	3.6	10,6
Germany	0.086	-0.140	-0.199	4.6	3.1	11,4
Greece	0.064	-0.052	-0.055	13.0	12.3	11,2
Italy	0.104	-0.150	-0.180	2.4	3.5	5,4
Japan	0.149	-0.245	-0.293	2.5	2.0	5,5
Korea	0.062	-0.199	-0.266	3.1	2.2	10,1
Mexico	0.102	-0.270	-0.450	2.3	1.2	12,6
Netherlands	0.085	-0.100	-0.145	6.6	4.4	6,6
Norway	0.066	-0.160	-0.184	4.0	3.4	12,1
Portugal	0.063	-0.072	-0.076	9.3	8.8	12,9
Singapore	0.029	-0.130	-0.193	5.0	3.2	1,1
Spain	0.063	-0.100	-0.182	6.6	3.5	11,1
Sweden	0.120	-0.083	-0.185	8.0	3.4	11,9
Switzerland	0.126	-0.120	-0.220	5.4	2.8	10,6
UK	0.117	-0.098	-0.141	6.7	4.6	10,4

*See note at bottom of Table 5.*

**Table 11**  
**Estimates of convergence speeds and conditional half-lives for RER - CPI-food**  
**(Pre-and Post-1987 (TAR model))**

Country	Threshold ( $\lambda$ )	$\rho_2$		CHL( $\rho_2$ )		p,m
		Pre-1987.3	Post-1987.3	Pre-1987.3	Post-1987.3	
Austria	0.14	-0.27	-0.29	2.2	2.1	8,5
Belgium	0.07	-0.20	-0.21	3.1	3.0	9,6
Denmark	0.06	-0.14	-0.19	4.6	3.3	12,3
France	0.135	-0.23	-0.26	2.7	2.3	11,6
Germany	0.11	-0.17	-0.19	3.7	3.3	10,6
Greece	0.08	-0.09	-0.12	7.3	5.4	11,4
HongKong	0.04	-0.23	-0.27	2.7	2.2	11,2
Japan	0.045	-0.24	-0.33	2.5	1.7	5,4
Korea	0.14	-0.28	-0.30	2.1	1.9	5,5
Netherlands	0.13	-0.26	-0.29	2.3	2.0	10,1
Norway	0.07	-0.11	-0.11	6.2	5.8	12,6
Singapore	0.02	-0.15	-0.16	4.3	4.0	6,6
UK	0.1	-0.26	-0.29	2.3	2.0	12,1