LIFE & DEATH

IN

PORTFOLIO THEORY

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In the opening sentence of his seminal 1959 monograph Markowitz wrote, "this monograph is concerned with the analysis of portfolios containing large numbers of securities …… a good portfolio is more than a long list of stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies". Yet in the 40 years of the subsequent development of modern portfolio theory the universe of securities from which optimum portfolios are constructed has scarcely gone beyond 1 risk-free bond and N risky stocks, hardly adequate to that "wide range of contingencies" which was Markowitz's original intent.

The purpose of this paper is to augment the universe of securities and include life and death contingencies into the framework of portfolio theoretic choice. The idea clearly is to investigate the place that life products occupy in the optimum portfolios of customers and to identify factors that influence the demand for them. It should be evident that the study of the choice of life products is amenable to the methods of portfolio theory because,

a) they satisfy the definition of "security" in the sense of being "prospects for a future receipt" [Sharpe (1970), pg. 79], and
b) they are risky in the sense of bearing a probability-weighted expectation of return with possibilities of not receiving the expected return.

Nothing more is required by portfolio theory. In particular, the theory does not require securities to be marketable – that is an impression created by a corollary of portfolio theory viz. capital market theory.

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Probability Distribution of Returns to Life Insurance

Our first task is to estimate expected returns and risks of life products. The probability distribution of the payoffs is as follows :

Year	**Probability**	**Payoffs**
1	d_1	A_1
2	d_2	A_2
3	d_3	A_3
:	:	:
:	:	:
m	d_m	A_m
--	*S*	A_S

where, $d_1, d_2, \ldots, d_m$ are investor's subjective probabilities of death in the policy years *1, 2, ….m*, and $s = 1 - \sum d_t$ is the subjective probability of surviving the policy term, $A_1, A_2, A_3, \ldots, A_m$ are the payoffs in the event of death in years *1, 2, …..m* and A_S is the survival payoff.

The table above is laid out in general form. Special cases include non-participating policies with fixed sum assured, $A_t = A$ for all *t*; participating endowment assurance policies with annual bonus, $A_t = A + tB$, pure term insurance policies $A_t = A$ and $A_S = 0$, pure endowment policies $A_t = 0$, $A_S = A$ and so on.

The probability distribution of the rate of return depends on the size and mode of premium payments and the benefits of the policy. If it is a single premium plan the rate of return r_t obtained in year *t* is the solution of

$$-P + \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \ldots\ldots\ldots + \frac{A_t}{(1+r)^t} = 0 \quad \text{viz. } r_{st} \text{ with probability } d_t. \quad (2a)$$

If it is a level annual premium the rate of return is the solution of

$$-P-\frac{P}{(1+r)}-\frac{P}{(1+r)^2}-\ldots\ldots-\frac{P}{(1+r)^{t-1}}+\frac{A_1}{(1+r)}+\frac{A_2}{(1+r)^2}+\ldots\ldots+\frac{A_t}{(1+r)^t}=0 \qquad (2b)$$

viz. r_{at} with probability d_t

Less obviously the table above can also be used to derive the probability distribution of returns on pension plans, the only difference being that A_t will now be annuities received. If C is the contribution then the rate of return in year t is the solution of

$$-C+\frac{A_1}{(1+r)}+\frac{A_2}{(1+r)^2}+\ldots\ldots\ldots.+\frac{A_t}{(1+r)^t}=0 \qquad (3)$$

r_{st} with probability d_t

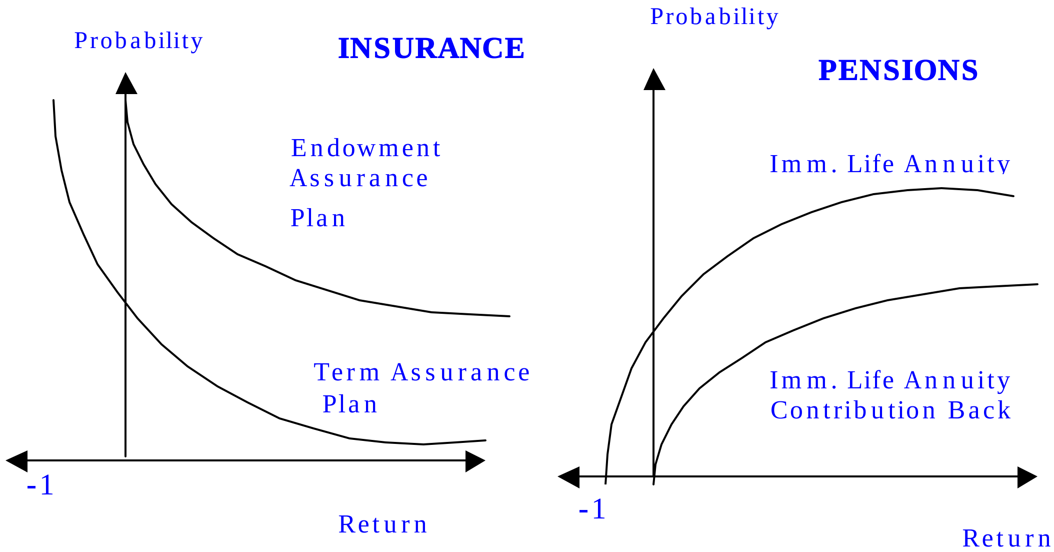
If the plan is a pure life annuity and if the individual dies in the first year the rate of return $r_1 = -1$ with probability d_1. If he dies in the second year he earns r_2, the solution of

$$-C+\frac{A_1}{(1+r)}+\frac{A_2}{(1+r)^2}=0$$ with probability d_2. And so on.

The expected return on the life product then is simply

$$\bar{r} = \Sigma d_t r_t + s r_S$$

where the return to survival r_s = -1 for a pure term plan or immediate life annuity, 0 for a premium refund plan and a positive number if it is an endowment assurance plan. The real difficulty lies in measuring risk. The variance of the rates of return will simply not do. That is because life products, both insurance policies and pension plans, exhibit highly skewed distributions of returns as the graphs below show :



The distribution of returns to insurance is skewed to the right – high returns with low probabilities of death in the initial years of the policy and low returns with the higher probabilities of death in later years. The distribution of returns to pensions is skewed to the left – low returns with low probabilities of death in the initial years of the plan and high returns with high probabilities of death (low probabilities of survival) in the later years. In these circumstances measuring risk by variance, which gives equal weightage to outcomes above and below the mean, will not be appropriate. Better measures would include :

a) Semivariance, i.e. variance below the mean rate of return

$S = \sum d_t (r_t - \bar{r})^2$ for $r_t \le \bar{r}$

b) Shortfall probability, i.e. the probability of not getting the mean rate of return

$p = \sum d_t$ for $r_t < \bar{r}$

c) The semivariance or shortfall probability measured from an external benchmark return, e.g. the risk free rate of return.

$\sum d_t (r_t - r_f)^2$ for $r_t \le r_f$ or $p = \sum d_t$ for $r_t \le r_f$

In what follows we shall adopt (a) the semivariance as the measure of risk on the grounds that it fits the formal constitution of portfolio theory better than the other measures. What can be more, it was suggested by Markowitz(1959) himself as a better though mathematically less tractable, measure as compounded to variance. For some applications it will also be useful to measure risk in the reverse sense by skewness using once again a measure suggested by Markowitz (1959)

$$K = \frac{\text{Variance}}{2\,(\text{Semivariance})} \tag{4}$$

Skewness gives an idea of the size of "upside opportunities" in relation to the "downside risk" (which is ironical but suggestive phraseology considering that upside may mean the high returns to death i.e. going up and downside may mean low returns to survival i.e. staying down). For a symmetrical distribution $K = 1$. Distributions skewed to right will have $K > 1$ and for distributions skewed to the left $K < 1$.

Liquidity of Life Products

In addition to return and risk we will also need to develop a measure for liquidity because both life insurance policies and pension plans come in a great variety in respect of the modes of premium payments, money back features, policy loans and surrenders as well as cash value withdrawal facilities. The liquidity that we wish to measure is not transactions liquidity in the sense of immediate encashability at low cost. We wish to develop a measure for the smoothness or lumpiness of the time pattern of cash inflows and outflows.

When comparing say a level premium endowment assurance plan and level premium money back plan the probabilistic duration of the plan can serve as a measure viz.,

$$D_U = \frac{\sum \frac{d_t A_t t}{(1+y)^t}}{\sum \frac{d_t A_t}{(1+y)^t}} \tag{5}$$

But when a level premium or a limited payment plan has to be compared with a single premium plan the measure above, which takes into account only cash inflows, will not do. The cash outflow pattern ought to be included as well. A measure of liquidity that incorporates the whole pattern of cash outflows and inflows is the mean term or duration of *net* cash flows.

$$D_N = \frac{\sum \frac{d_t A_t t}{(1+y)^t} - \sum \frac{P_t t}{(1+y)^t}}{\sum \frac{d_t A_t}{(1+y)^t} - \sum \frac{P_t}{(1+y)^t}} \tag{6}$$

It is readily verified that the duration D_N of a portfolio is the weighted sum of the durations of the securities in the portfolio. The measure holds good in the domain of the viability of the life product from the viewpoint of the customer i.e. for those subjective probabilities of death for which the present value of benefits exceeds the present value of premiums paid.

Covariance Of Returns

The covariances between rates of return of different life products are also readily found. Consider two products A & B and their distributions.

	A	***B***
d_1	r_{A1}	r_{B1}
d_2	r_{A2}	r_{B2}
d_3	r_{A3}	r_{B3}
:	:	:
:	:	:
d_m	r_{AM}	r_{BM}
s	r_{AS}	r_{BS}

The covariances between returns are

$$\sum d_t (r_{At} - \bar{r}_A)(r_{Bt} - \bar{r}_B) + s(r_{As} - \bar{r}_A)(r_{Bs} - \bar{r}_B)$$

And the semicovariance is the covariance of r_{AT} and r_{BT} in the domain $r_{At} \leq \bar{r}_A,\ r_{Bt} \leq \bar{r}_B$. This expression will hold for all products having the same maturity. Clearly, the covariances of life insurance policies among themselves will be positive as will those of pension plans but the covariances between life policies and pensions will be negative.

Unit linked life products whose 'savings element' is invested in a specified portfolio stocks will have some peculiarities. Firstly, the expected return the unit linked plan will be the sum of the expected return on the mortality component and the expected return on the portfolio i.e.

$$E(r_U) = E(r_L) + E(r_p)$$

Since the mortality risk and portfolio risk are independent of one another the risk of unit linked plan too will be a simple summation of the two risks.

$$V(r_U) = V(r_L) + V(r_p)$$

Clearly the returns on unit linked products with the index portfolio will exhibit significant correlation. If the beta of the portfolio chosen by the unit linked plan with the index portfolio is known then

$$Cov\ (r_U\ ,\ r_m) = b_p\ V(r_m)$$

and of course,

$$V(r_p) = b_p^2 V(r_m)$$

and if the chosen portfolio is itself the market index i.e. $p = m$

$$Cov\ (r_p,\ r_m) = V(r_m)$$

and

$$V(r_U) = V(r_L) + V(r_m)$$

and

$$E(r_U) = E(r_L) + E(r_m)$$

Portfolio Equations

Let us now proceed to construct portfolios under varying general conditions from the following universe of assets; (a) a low return low risk fixed deposit, a medium return medium risk income scheme, a high return high risk index of stocks, and a set of life insurance policies and (b) the first three assets of (a) and a set of pension plans. We will report the results by way of simulations in three parts. In the first part we seek optimum risk return portfolios obtained from

$$\text{Minimise } Z = -l_1 E_p + S_p \text{ subject to } \sum x_i = 1 \quad (7)$$

where E_p, S_p and x_i are the portfolio return portfolio semivariance and proportions of wealth invested in security i respectively and $l_1 = \frac{\partial S_p}{\partial E_p}$ is the coefficient of risk tolerance and can take values between 0 and ∞. In the second part we optimize return, risk and liquidity by

$$\text{Minimise } Z = -l_1 E_p + S_P + l_2 D_p \text{ subject to } \sum x_i = 1 \quad (8)$$

where D_p is the duration of the portfolio and $l_2 = \frac{\partial V_p}{\partial D_p}$ is the coefficient of illiquidity tolerance in the following sense – it measures the decrease in risk that the investor would require to tolerate an additional unit of illiquidity, or equivalently, the additional risk he would be willing to tolerate for an additional unit of liquidity.

Some considerations on which the simulations are based are as follows :

a) We shall assume that life and death contingencies are independent of bond and stock return distribution i.e. covariances between life products and other assets are zero (the few suicides that take place in stock market crashes are in any case not eligible for insurance claims).

b) The duration of *net* cash flows for stocks and bonds is zero

c) The results of the simulations are based on variations in (i) subjective probabilities of death (ii) changes in the coefficients of risk and illiquidity tolerance (iii) returns and risks of other assets.

At every stage we find the desired asset allocation x_i as solved from the usual Markowitz system of equations :

$$\begin{bmatrix} 2S_{11} & 2S_{12} & \ldots\ldots & 2S_{1n} & -1 \\ 2S_{21} & 2S_{22} & \ldots\ldots & 2S_{2n} & -1 \\ : & : & \ldots\ldots & : & : \\ : & : & \ldots\ldots & : & : \\ 2S_{n1} & 2S_{n2} & \ldots\ldots & 2S_{nn} & -1 \\ 1 & 1 & \ldots\ldots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_n \\ m \end{bmatrix} = \begin{bmatrix} l_1 R_1 - l_2 D_1 \\ l_1 R_2 - l_2 D_2 \\ : \\ : \\ l_1 R_n - l_2 D_n \\ 1 \end{bmatrix}$$

where the S_{ij} *(i ¹ j)* are the semicovariances, S_{ii} are the semivariances and µ is the Lagrange multiplier associated with the wealth constraint $Sx_i = 1$.

The plan of reporting the simulations is as follows. We have not considered all policies together. This is necessary for the reason that the correlation coefficients and semicorrelation coefficients between plans are very high as shown below which causes numerical difficulties viz. when all plans are considered together, there are difficulties of matrix inversion and the numerical results become unstable. Also portfolios containing life covers and annuities are reported separately. But this has a formal reason. Customers purchase life plans only if their subjective mortalities of death are *greater* than the mortality used by the life insurers in the premium calculation. Also customers purchase annuity plans only if their subjective mortality rates are *lower* than the mortality used by the insurer to calculate the contribution. Observe that these conditions are contradictory. Hence it is best to keep insurance plans separate from pension plans even though due to negative correlation coefficients between insurance plans and pension plans portfolios containing both will be extremely attractive.

Data

The data used in the simulations are as follows. We consider a man aged 35 considering a set of life insurance plans of maturity 15 years. The plans include :

1. Single premium with profits endowment assurance plan
2. Annual premium money back with profits (with additional cover)
3. Annual premium with profits endowment assurance plan
4. Annual premium, premium back term assurance plan

The average rate of return, variance, semivariance, skewness and the durations of gross and net cash flows are shown below on 3 subjective mortality assumptions. We have assumed that the latest bonus rates will be maintained throughout the terms of the plans

a) Low – equal to insurer's assumed mortality
b) Medium – equal to 3 times (a)
c) High - equal to 5 times (a)

PLANS (SUBJECTIVE MORTALITY LOW)

	1	2	3	4
Average Return	0.072	0.076	0.103	0.127
IRR (Survival)	0.070	0.055	0.074	0.000
Variance	3×10^{-4}	0.099	0.276	6.95
Semivariance	2.84×10^{-6}	4.1×10^{-4}	7×10^{-4}	0.016
Skewness	57.70	120.73	175.44	222.74
Duration (U)	14.73	10.41	14.72	14.92
Duration (N)	2210.50	180.65	1180.55	62.90

PLANS (SUBJECTIVE MORTALITY MEDIUM)

	1	2	3	4
Average Return	0.075	0.117	0.1601	0.3824
IRR (Survival)	0.070	0.055	0.0746	0.000
Variance	9.67×10^{-4}	0.295	0.8228	20.77
Semivariance	2.38×10^{-6}	0.0003	0.0067	0.1307
Skewness	20.26	43.40	61.27	79.482
Duration (U)	14.21	10.250	14.18	12.9
Duration (N)	720.26	65.32	379.72	26.38

PLANS (SUBJECTIVE MORTALITY HIGH)

	1	2	3	4
Average Return	0.079	0.158	0.2171	0.637
IRR (Survival)	0.070	0.055	0.074	0.000
Variance	0.0015	0.4874	1.363	34.46
Semivariance	6.19×10^{-5}	0.0086	0.017	0.345
Skewness	12.77	28.29	38.14	49.86
Duration (U)	13.70	10.19	13.66	12.09
Duration (N)	422.21	45.19	219.56	19.07

Some observations can be made immediately. Firstly risk oriented plans have greater variances and semivariances as compared to investment oriented plans. Secondly, risk oriented plans e.g. Plan 4 are characterized by greater skewness (variance in relation to semivariance) as compared to investment-oriented plans. E.g. Plan 1. Thirdly, risk oriented plans have larger returns to death as compared to return to survival. Fourthly, money back plans have the highest degree of liquidity as compared to other plans. But a level premium plan will show greater liquidity compared to a single premium plan. Observe that the duration of net cash flows captures this better than the usual duration. Finally as the subjective mortality rate increases the skewness declines across all plans and so do the durations, both usual and net. But the rate of change is not uniform across plans, e.g. the skewness of plan 4 declines from 222.74 at low mortality to 49.86 a drop to 22.3% but that of the plan 2 declines from 120.73 to 10.19 a drop to 8.44%. Likewise the response of duration to changes in subjective mortality is not uniform either.

Next consider the semicorrelation coefficient between the policies. (these have been calculated using 12 plans instead of the 4 that have been reported in the simulations)

PLANS (SUBJECTIVE MORTALITY LOW)

	1	2	3	4	5
1	1	0.9976	0.0085	0.9987	0.9976
2	0.9976	1	0.9962	0.9963	0.9897
3	0.0085	0.9962	1	0.9999	0.9962
4	0.9987	0.9963	0.9999	1	0.9963
5	0.9976	0.9897	0.9962	0.9963	1

PLANS (SUBJECTIVE MORTALITY HIGH)

	1	2	3	4	5
1	1	0.9820	0.9594	0.9855	0.9888
2	0.9820	1	0.9855	0.9855	0.9756
3	0.9594	0.9847	0.9888	0.9888	0.9961
4	0.9855	0.9603	0.9987	1	0.9951
5	0.9888	0.9756	0.9961	0.9951	1

The semicorrelation coefficients (as well as the usual correlation coefficients not reported here) are very high but tend to weaken as subjective mortality rates increase.

The correlation coefficient between variance of policy returns with their semivariance fluctuates non-monotonically as subjective mortality rises.

Mortality	Correlation Coefficient
1 Time	0.380
2 Times	0.707
3 Times	0.961
5 Times	0.868

This is also true of the correlation coefficients between the covariances and semicovariances. This, apart from the fact that the distributions are skewed, is an additional reason for using semivariances and semicovariances.

Life Insurance : Return – Risk

We first report the results of including two life insurance plans, an investment oriented single premium with profits endowment assurance plan and a risk oriented term assurance premium back plan alongside other assets. The following assumptions have been made for the simulations reported below. The return on fixed deposit is 6.5% with a semicovariance of 0.01%, the income scheme gives an 11% return with a semivariance of 0.1% and the expected return on stocks in 19% with a semivariance of 0.8%. The semicovariance between the fixed deposit and the income scheme is set at 0.011%, the semicovariances between the fixed deposit and the income scheme vis-à-vis stocks and life insurance plans are assumed to be zero. These are only a tentative set of figures. We shall vary these figures in later simulations to observe their impact on the proportions in the portfolio of the various assets. The idea is not to estimate actual magnitudes of assets demanded, the idea is to see how the proportions vary due to various environmental changes such as the subjective mortality rates, the premiums charged, the coefficient of risk tolerance, and the returns/risks of the assets.

SUBJECTIVE MORTALITY LOW

	l_1		
Asset	**0.025**	**0.04**	**0.05**
Fixed Deposit	-1.512	-2.421	-3.027
Income Scheme	0.647	1.036	1.295
Stocks	0.185	0.296	0.370
Endowment	1.657	2.044	2.302
Term	0.022	0.44	0.058

SUBJECTIVE MORTALITY MEDIUM

	l_1		
Asset	**0.025**	**0.04**	**0.05**
Fixed Deposit	-1.510	-2.418	-3.023
Income Scheme	0.647	1.036	1.295
Stocks	0.185	0.296	0.370
Endowment	1.670	2.065	2.329
Term	0.007	0.019	0.028

SUBJECTIVE MORTALITY HIGH

	λ_1		
Asset	**0.025**	**0.04**	**0.05**
Fixed Deposit	-1.507	-2.420	-3.029
Income Scheme	0.647	1.036	1.295
Stocks	0.185	0.296	0.37
Endowment	1.676	2.082	2.353
Term	-0.001	0.005	0.009

Three observations may be made. Firstly as risk tolerance increases, the proportions of risky assets i.e. stocks and the term insurance plan rise rapidly as compared to the less risky assets viz. the fixed deposit and the income scheme. Secondly, as the subjective mortality rate decreases the proportion of term insurance plans is seen to rise, the proportion of safe endowment assurance plans falls and proportion of stocks is seen to increase marginally. Observe that this is a counterintuitive result – prima facie one is tempted to infer that those who fear early death will buy cheap term insurance to obtain high returns. But it doesn’t happen that way. As subjective mortality rate increases the average return of term plans does rise but the risk rises even faster unlike endowment assurance plans. This explains the result

Identical results are obtained if we substitute the single premium endowment assurance plan with level premium endowment assurance plan or a money back plan.

Other variables can be changed to study their impact on the portfolio. An increase in the return on the fixed deposit from 6.5% to 8.5% has the following effect.

LOW SUBJECTIVE MORTALITY

	λ_1		
Assets	0.025	0.04	0.05
Fixed Deposit	1.323	2.116	2.644
Income Scheme	0.336	0.538	0.676
Stocks	0.185	0.296	0.370
Endowment	-0.902	-2.05	-2.816
Term	0.056	0.099	0.127

The investment oriented single premium endowment assurance plan's share declines steeply due to the rise in the interest rate but the share of the term plan rises, i.e. the endowment assurance plan is substituted on the investment side by the fixed deposit and on the insurance side by the term assurance plan.

Likewise, an increase in the return on stocks causes its own share to increase and the shares of all other assets to decline. An increase in the risk of stocks has the opposite effect. However, when stock returns become very high the share of life insurance plans show an increase at very high risk tolerance levels ($\lambda_1 \geq 1$). This happens due to the zero correlation between life plans and stocks which moderates overall portfolio risk when life plans are bought.

Life Insurance : Return, Risk and Liquidity

Next, consider the same question in 3-dimensional portfolio theory i.e. in terms of return, risk and liquidity. Since liquidity measures of the assets involved are very disparate we shall confine ourselves to two life insurance policies. Viz. single premium endowment assurance and the level premium assurance, the former being more illiquid (less liquid) as compared to the latter. We shall ignore other assets for the moment. To report only small cross section of the results suppose λ_1 = 0.04 and $l_2 = \frac{\partial V_P}{\partial D_P}$ takes on the different values as shown below :

LOW SUBJECTIVE MORTALITY

λ_2	-1.5×10^{-5}	-3×10^{-7}	-7.5×10^{-8}
Endowment (Single Prem.)	-10.94	-0.04	0.118
Endowment (Annual Prem.)	11.94	1.04	0.881

As the coefficient of illiquidity tolerance decreases, (i.e. the investor is satisfied with a smaller reduction in risk to tolerate an additional increase in illiquidity) we observe a tendency for the proportion of the illiquid plan (single premium) to rise and that of the liquid plan (annual premium) to fall. Identical results are obtained if instead of the level annual plan, we substitute the limited payment plan or the money back plan.

At higher levels of subjective mortality the share of the relative illiquid plans is greater as compared to the liquid plans. In other words at low mortality rates, individuals tend to prefer liquid plans.

HIGH SUBJECTIVE MORTALITY

λ_2	-1.5×10^{-5}	-3×10^{-7}	-7.5×10^{-8}
Endowment (Single Prem.)	0.7915	0.885	0.886
Endowment (Annual Prem.)	0.2085	0.114	0.113

This somewhat enigmatic result establishes the idea that liquidity is considered by customer to be a "living benefit".

Pensions

The behaviour of the return and risk of pension plans with respect to the subjective mortality rate is very different from that of life insurance. In the case of life insurance a decrease in subjective mortality causes both the expected returns and risks to decline. In case of pensions a decline in subjective mortality increases the expected returns but the risks decline. Rising returns and falling risks makes pensions increasingly attractive at lower subjective mortality rates.

The table below shows the data of two typical pension plans that are sold commercially in India, (i) Immediate life annuity (ii) Immediate life annuity with contribution back on death. There are other plans but they are minor variants of these basic types. Yet others, e.g. government pension plans are administered and may or may not be based on meticulous actuarial calculation.

We consider a man aged 50. The figures are shown at 3 subjective mortality rates (a) Low - equal to insurer's assumptions, (b) Medium – 1.5 times of (a) and (c) high – 2.5 times of (a).

LOW – SUBJECTIVE MORTALITY

	1	2
Average Return	0.1060	0.1255
Variance	0.0151	1.04×10^{-4}
Semivariance	0.0145	1.00×10^{-4}
Skewness	0.5230	0.5186
Duration	13.77	13.07

MEDIUM – SUBJECTIVE MORTALITY

	1	2
Average Return	0.0903	0.1241
Variance	0.0220	1.5 X 10-4
Semivariance	0.0206	1.43 X 10-4
Skewness	0.5339	0.5314
Duration	13.07	13.07

HIGH SUBJECTIVE MORTALITY

	1	2
Average Return	0.0528	0.1143
Variance	0.0342	2.92×10^{-4}
Semivariance	0.0306	1.78×10^{-4}
Skewness	0.5588	0.8179
Duration	11.552	11.183

The concept of duration of net cash flows does not apply to the two pension plans considered here because there are no variations whatsoever in time patterns of cash flows; in both plans a single contribution is made upfront and the annuity begins to flow until death. If, however, we were to considered deferred annuity plans the net durations would have to be calculated.

The covariances and semicovariances between the plans at the three subjective mortalities are

	Low	Medium	High
Correlation Coefficient	0.9512	0.9496	0.8367
Semicorrelation coefficient	0.9495	0.9466	0.9053

In this respect pension plans resemble life insurance plans; at higher subjective mortality rates the correlation coefficient decline.

It is clear from the data that the contribution back scheme dominates the immediate life annuity in all dimensions; return, risk and liquidity. Accordingly in the simulations reported below we shall consider only plan 2 along with the substitute assets.

LOW – SUBJECTIVE MORTALITY

	λ_1		
Assets	**0.025**	**0.04**	**0.05**
Fixed Deposit	-4.06	-6.80	-8.63
Income Scheme	0.675	1.083	1.35
Stocks	0.158	0.243	0.301
Pension (2)	4.23	6.48	7.98

MEDIUM SUBJECTIVE MORTALITY

	λ_1		
Assets	**0.025**	**0.04**	**0.05**
Fixed Deposit	-2.74	-4.75	-6.09
Income Scheme	0.660	1.06	1.32
Stocks	0.170	0.267	0.332
Pension (2)	2.91	4.42	5.43

HIGH – SUBJECTIVE MORTALITY

	λ_1		
Assets	**0.025**	**0.04**	**0.05**
Fixed Deposit	-2.36	-4.17	-5.37
Income Scheme	0.656	1.055	1.32
Stocks	0.174	0.274	0.341
Pension (2)	2.53	3.84	4.71

Observe that as subjective mortality declines the demand for pensions rises rapidly at the cost of the fixed deposit. The slightly riskier income scheme's share is seen to improve slightly and the share of stocks goes down a little. These latter do not show high sensitivity. If we simulate without the income scheme, however, the demand for stocks is seen to increase as subjective mortality declines. Of course with respect to the coefficient of risk tolerance, as it increases the demand for the stocks is seen to rise much faster as compared to any of the other assets as can be verified by the percentage increases. The sensitivity of the portfolio with respect to changes in the interest rate and the return and risk of stocks can be worked out in this case as well. An increase in the return on stocks causes the shares of stocks and the pension plan to increase at the cost of the fixed deposit and the income scheme.

Analysis

Having taken glimpses of partial cross sections of a multidimensional phenomenon, it's time to take stock of the key results that we have obtained so far.

1. A decline in the subjective mortality rate
   a) causes the demand for life insurance to decline and the demand for pensions to rise, which is self-evident,
   b) causes a shift in the life insurance product mix in a not-so-obvious way; the demand shifts in favour of term assurance or double/triple cover plans and away from endowment assurance and money back plans
   c) causes the demand for liquid plans to rise in relation to illiquid plans

2. An increase in the coefficient of risk tolerance
   a) causes the demand for risky assets like stocks to rise
   b) causes a shift in the desired life product mix towards risk oriented term assurance plans away from investment oriented endowment assurance and moneyback plans.

3. An increase in the coefficient of illiquidity tolerance
    a) causes the demand for relatively illiquid plan like single premium or limited payment plans to rise in relation to level premium or money back plans
    b) causes a shift away from such product features as early surrenders or loans towards plans that may package greater risks (and returns) with greater illiquidity.

4. A change in the rate of interest
    a) has a large effect on safe policies like endowment assurance plans and money back plans in the opposite direction
    b) has in comparison with the above, an insignificant impact on risk-oriented plans like term insurance or double/triple cover endowment plans.

Experience

It would be unreasonable to expect that direct empirical evidence in terms of the precepts of modern portfolio theory would be available in readymade form. . There are, besides, the difficulties of finding proxies for evanescent concepts such as the subjective mortality rate, the coefficient of risk tolerance and the coefficient of illiquidity tolerance.

Let us make two working hypotheses that can force the inferences to have operational significance.

1) We shall suppose that subjective mortality rates (individuals' anxieties and fears regarding early demise) are directly related to objective mortality experience of a society
2) We shall also suppose that the coefficients of risk and illiquidity tolerance are directly related to wealth/income of individuals, i.e wealthier an individual the more his tolerance of risk[1].

[1] This assumption actually violates modern portfolio theory. See Note below.

Consider now the following data. Table 1 shows the world distribution of annuities, life and health insurance business from 1984 and 1996

Table 1

Distribution of Business

	1984	1996
Life	38	29
Health	30	23
Annuities	32	48

Observe the marked decline in life insurance business and the rise in annuities business in relatively small span of 12 years.

Consider the data for USA. In the USA the percentage of life insurance sale in total business of life insurance companies has fallen from 69.1% in 1960 to 28.4% by 1997. During the same period the percentage of annuities rose from 7.7% to 48.7%. Within life insurance the product mix has shifted dramatically as figures below show :

Table 2

Year	% Share of New Business (USA)		
	Term	Whole Life / Endowment	Universal / Variable
1980	57%	42%	-
1990	48%	23.5%	28.5%
1994	48%	16.7%	35.3%

Universal/Variable life plans are highly flexible plans that give flexible investment options to policyholders as well cash value withdrawal options. These are high return, high risk, high liquidity plans. In the case of pensions too there is a shift in product mix towards variable annuities. Between 1979 to 1995 the number of variable annuity holders rose from 2 million to 12.8 million and its share in total annuity business rose from 10.7% to 24.5%.

Table 3 shows the results of the annual surveys conducted by LIMRA of their customers in the US.

Table 3

	% of people believing that most people must have life insurance*	% of consumers who view life insurance favourably*
1968	-	72%
1980	75%	-
1988	68%	60%
1992	-	-
1994	-	35%
1996	52%	-

*(% having no strong opinion rose from 12% to 33%)
American Council of Life Insurance 1998 (USA)

Table 4 shows the new business premium in UK. The largest growth recorded is of unit linked plans with facilities for repurchase of units..

Table 4

	% share in Total Premium	
Year	Annual Premium	Single Premium
1965	88.52	11.48
1975	76.80	23.20
1985	25.50	74.00
1990	29.83	70.17
1994	12.64	87.36

Association of British Insurers : Insurance Statistics for various years : Insurance Fats, Figures and Trends, September 1998

Table 5 shows the distribution of life business in Germany. It brings out the fact that endowment assurance business has declined rapidly and is being by substituted by term assurance plans while group insurance business shows steady figures.

Table 5

	Endowment	Tax Saving	Term	Annuities	Group
1950	88.1		3.0	1.0	7.9
1960	80.3		7.4	4.2	8.1
1970	75.3		10.6	4.2	9.9
1980	64.3	1.9	15.4	2.8	15.6
1990	67.7	1.6	16.4	4.9	9.4
1993	54.4	1.4	22.2	12.6	9.4

Germany : Insurance Research Letter – April 1988 & February 1995
Tillinghast – Towers Perrin : Insurance Pocket Book

Japanese data show that the percentage of term and whole life plans stands at 52% and unit linked business is more than 50% of the endowment assurance business. Japan offers an example of the high interest sensitivity of life insurance sales. Between 1995 to 1997 the real growth rate of life insurance sales fall from 5.4% to –2.7%, individual annuities from 47.9% to –5.5% and of group life insurance from 3.8% to –2.7%. Incidentally Indian experience for 2000 – 2001 are also indicative of this phenomenon. As interest rates fell (e.g. Public Provident Fund rate from 11% to 9.5%) life insurance sales rose by a record of 35% in a single year. And the great wave of American Life insurance insolvencies due to Reagan's tight money policies which saw the money market rates rise from 8 – 10% to 18 – 20% in a period of 3 months is part of the folklore of insurance. (Japan Institute of Life Insurance, Japanese Life Insurance Association : quoted in Japanese Insurance News March/April 1994 for 1965-91. Data for 1993-97 from JILI. Life Insurance Fact Book 1997, SIGMA Prospect No. 6 / 1998

Next consider the data on the Australian market. Table (6) shows the distribution of life insurance business in the years

Table 6

	1992		1997	
	Annual Premium	Single Premium	Annual Premium	Single Premium
Whole Life & Endowment	4.7	0.05	1.9	0.1
Term	6.2	0.1	13.1	0.1
Accident/Disability	5.3	-	10.4	-
Unit Linked/ Superannuation	48.6	47.1	51.8	55.3

Australia : ISC Quarterly Statistical Bulletin – quoted in Insurance Pocket Book; Tillinghast, NTC Publication

During the period 1971-1995 share of life insurance premiums declined from 68% to 33% and the share of annuities rose from 32% to 67%. The share of single premiums plans rose from 5% to 92% and that of annual premiums plans fall from 95% to 8%.

In Canada between 1980 – 1997 the share of life insurance fall from 39% to 27.9% but the share of annuities grew from 39% to 50%. (Canada Life & Health Insurance Facts : Report of Superintendent of Financial Institutions, Ottawa)

Remarks

The concordance between the inferences from the theory and the actual experience of the developed countries just cited is notable. Life expectancies in the developed world have improved dramatically during the last three decades and stand in the region of 90 to 100 years for the countries whose data has been reported. Simultaneously the wealth and per capita income in these countries are high. Our analysis is suggested that these are precisely the forces that shift consumer preferences towards risk oriented term insurance plans, high return high risk stocks, plans having greater liquidity even as capacity to pay single premiums increases and annuities with variable features. This has caused the product mix of life insurers to change so dramatically and resulted in a new technology for packaging return, risk and liquidity. To be sure there has been an impetus from the supply side as well with unprecedented increases in the volatilities in financial markets. Life insurers have tended to shift away from guaranteed return contracts preferring to pass on the investment risk to their customers.

Of course, all this is still in stark contrast with the experience of the developing world where traditional endowment assurance and money back plans do more than 85% of life insurance business and commercial pensions markets hardly exist. Only some of the trends that have firmly manifested in the developed countries will show themselves up in the developing countries, most notably a rise in financial markets' volatility and fierce competition in the life insurance market place. This is bound to impact product design from the supply side. But from the demand side the response is likely to be slow on account of the slow progress on the growth of income / wealth and life expectancy. It may be conjectured that it will largely result in finer market segmentation with pure term assurance and/or unit linked plans being sold to high income sections and traditional plans continuing amongst the middle and low income sections.

Note 1

Wealth and Risk Tolerance

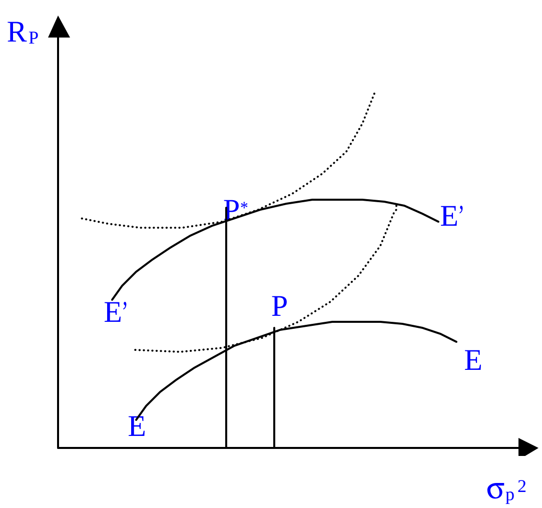
It should be pointed out that the direct relation of wealth and risk tolerance posited by us for bringing a correspondence between the analysis and evidence is actually at variance with the tenets of modern portfolio theory. Recall that this theory uses a quadratic utility function.

$$U_p = a + bR_p - cR_p^2 - c\,\sigma_p^2$$

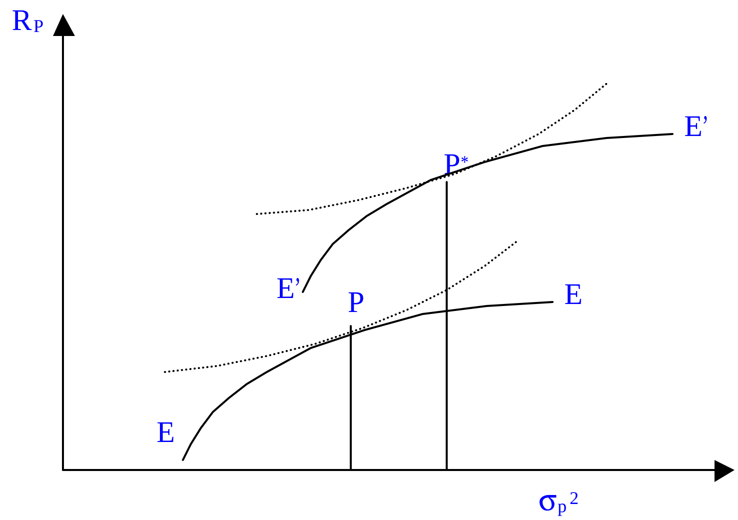
so that the indifference curves are circles with centers on the return axis. In equilibrium

$$\frac{\partial V_p}{\partial R_p} = \lambda_1 = \frac{b - 2cR_p}{c}$$

Now the property of this equilibrium is that if the efficient frontier is lifted up due to higher returns at every level of risk (i.e. the individual becomes richer) the new equilibrium will be at a point where he prefers to take lesser risk than before.



In the diagram EE shows the efficient frontier and the dotted lines are indifference curves. As the diagram shows in the new equilibrium P^* the level of risk is lower. This is the most controversial prediction of modern portfolio theory and most people think that this behaviour is inconsistent with reality. So the alternative type of situation envisaged in this paper is to posit that as wealth increases the utility function itself undergoes a change in the parameters b and c, i.e. b increases and c decreases so that l_1 increases and the resulting situation is as shown below :



The indifference curve corresponding to a greater wealth shifts northeast and causes the proportion of risky assets held to rise.

Note 2

Literature

The theory of life insurance demand is an underresearched are. Huebner (1964) developed the widely acclaimed idea that the life insurance required by an individual should be equal to his human life value, i.e. the present value of future earnings minus own consumption over the remaining life time. This idea is based on the indemnity principle of general insurance. But although widely acclaimed and indeed an integral part of any insurance agents' toolkit this model is neither used and sell nor buy life insurance.

Most of the subsequent development of this theory has taken place in the context of a life cycle model of saving; the consumer is assured to maximize life time expected utility given that he will earn an income stream y(t) and incur a consumption stream c(t). His savings are,

$$S(t) = \int_0^{\infty} e^{-d\,s}[y(s) - C(s)]\,ds$$

where δ is the force of interest. The function may be constrained in several ways, e.g.

(i) S(T) ≥ B, i.e. the consumer leaves a bequest B at the time of death

(ii) S(t) ≥ 0 i.e. the consumer is never indebted in net terms

(iii) S(T) ≥ 0 i.e. the consumer is solvent at time of death, etc.

Given y(t) he maximizes utility to determine a consumption plan c(t) which depends on the utility function chosen. A function that has been popular in this literature [Yaari(1963,1965), Hakansson (1969), Fischer (1973), Richard (1975), Borch (1977)] is

$$U(c) = \int_0^{T} e^{-b\,s}\; u[c(t)]\,dt$$

where β is the "impatience to consume". The solution is,

$$c(t) = K\,e^{(l-d)t}$$

Life insurance is then brought in by introducing the probability death π(t) so that the expected utility of consumption viz.,

$$\int_0^T e^{-bt}\{p(t)u[c(t)+y(t)]+(1-p(t)]u[c(t)]dt$$

which is maximized subject to one of the constraints above.

The role of life insurance in these models is to (a) smooth out consumption, (b) to make bequests (c) to repay debt or (d) to draw a pension. The only alternative to life insurance buying is to borrow or lend at the risk free rate to smooth out life time consumption. Special results are brought out by Hakansson (1969) who investigates the bequest motive, Lewis (1989) finds the influence of the number of dependents on the demand for insuring the life of the breadwinner, Richard (1975) demonstrates model that explains the inverse relation of the wealth of the individual and his demand for life insurance, etc.

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