

Measuring Mutual Fund Performance Using Lower Partial Moment

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Abstract

In this paper, we have developed measures of evaluating portfolio-performance based on LPM (Lower-Partial-Moment). The three traditional measures by Treynor, Sharpe, and Jensen are based on the Mean-Variance (M-V) rule, which is valid only when the distribution of asset returns is characterized by spherical symmetry to which class normal and similar distributions belong. But, Bawa has proved that the M-LPM₂ (Mean - LPM of 2nd Order) rule can be used as a reasonable approximation of the optimal rule for the entire class of return distribution. Risk from the LPM perspective is measured by taking into account only those states in which return is below a pre-specified "target rate", like risk-free rate, and capturing the extent to which it is below. We also provide a new way to evaluate the performance of a portfolio, which is similar to the M² [Modigliani-Modigliani] approach, but differs from it in an important way.

Lower Partial Moment and Mutual Fund Performance

In the utopian world of perfect capital markets, all investors would put part of their money in the risk-free asset and the other part in the market-portfolio, the unique optimal risky portfolio consisting of all assets. Since there is no transaction costs and no informational asymmetry, there is no reason for anyone to hold a portfolio other than the market portfolio.

But, in the real world, there are transaction costs and there is informational asymmetry. People hold different portfolios, either directly or indirectly through portfolio-managers and mutual-funds. Since the market portfolio contains all securities, these individualized portfolios are "subsets" of the market portfolio. Different subsets include different securities, and, therefore, the weight of a particular security in each subset is different from what it is in market portfolio. In any case, once one deviates from the market portfolio, the question arises as to what is the best "subset" of the market-portfolio that one should hold.

The most popular practical market portfolio is a portfolio mimicking a broad-based index, like the S&P 500, Willshire 5000, Russell 4500, or NYSE Composite Index in USA. Since it is immensely difficult to construct the theoretically optimal market portfolio, such an index is considered to be the surrogate for it. Thus, this becomes the benchmark with respect to which all other broad-based portfolios are evaluated. And, this is what is popularly called "the market portfolio". Similarly, there are sector or industry indices with respect to which sector/industry-based portfolios (portfolios investing in a specific sector or industry) are judged. If we believe that the capital-market is efficient, however, no portfolio should be doing better than the market-portfolio. In many instances, the purpose of evaluating different portfolios is indeed to check whether this holds true. If a portfolio performs better than the market, we say that it has "beaten" the market. If a portfolio "beats" the market consistently, that points toward inefficiency of the capital-market in some form. On the other hand, if no portfolio consistently beats the market, not only does it corroborate the efficiency of the capital-market, it also questions the wisdom of paying fees to a portfolio-manager or a mutual-fund for managing one's portfolio. In an efficient capital-market, the best strategy is to put the money in the market portfolio and passively watch it. There is no role for active portfolio management in such a world.

In earlier days, performance of portfolios involved comparing their returns. That implicitly assumed that investors do not care about the risk of the portfolio they hold. But, it was soon recognized that investors care about both the return and risk of their portfolio, as both these enter their utility functions. One could often earn a higher return by taking a higher risk, and that should not be taken as superior performance. Therefore, it became imperative that one compares the risk and return of a portfolio to that of the market. Since this two-dimensional comparison is to be ultimately brought to one dimension, there emerged the concept of "risk adjusted" performance. This involves comparing not the raw returns, but the abnormal-returns, that is the returns net of the RRR (required-rate-of-return), where the RRR is a function of the risk.

The three traditional measures to evaluate performance of portfolios have been enunciated by Treynor (1965), Sharpe (1966), and Jensen (1968). Treynor measure takes the excess-return on a portfolio (measured as the raw return minus the corresponding risk-free rate) and divides it by the beta of the portfolio. Sharpe measure in contrast divides the excess-return by the standard-deviation of the portfolio. Thus, whereas Treynor measure focuses on the systematic or non-diversifiable risk of a portfolio, Sharpe measure focuses on the total risk as measured by the standard-deviation or sigma. Jensen's measure is different from these two in that it focuses on the abnormal-return, return on a portfolio in excess of its RRR; it, of course, uses the standard CAPM (Capital Asset Pricing Model) to compute the RRR.

More recently, Modigliani and Modigliani (1997) have provided a new perspective. Their approach is similar to the Sharpe measure and gives the same result. But, they follow a novel approach for evaluating a portfolio vis-à-vis the market-portfolio. They combine the portfolio to be evaluated with the risk-free asset to create a new portfolio having the same standard-deviation (or sigma) as the market-portfolio. Then they go ahead and compare the return on this new portfolio to that of the market-portfolio.

As we know, all the above measures are based on the CAPM, which is based on the mean-variance framework. It is therefore important to recognize the edifice upon which the CAPM is built and its implication for portfolio evaluation.

Choosing among portfolios is basically choosing among probability distributions. Von Neuman and Morgenstern (1967) have characterized this choice problem as one of utility maximization. In the absence of complete information about individuals' utility functions, however, the best we can do is to delineate the set of admissible alternatives from among the given alternatives, by throwing away those alternatives that are inferior for a given class of utility function. One would, of course, seek an optimal rule that minimizes the admissible set. On the one hand, a smaller admissible set would be useful from a practical perspective. But, on the other hand, this would require more restrictions on the class of utility functions, which, in turn, would result in the loss of its generality. A good compromise is to consider the most restrictive class of utility functions that is suggested by observed individual behavior. Pratt (1964) and Arrow (1971) have put forth DARA (decreasing absolute risk aversion) and, to certain extent IRRA (increasing relative risk aversion), as characteristic of individual behavior. But, since Stiglitz (1970) has criticized the plausibility of IRRA, DARA seems to be the proper restrictive class.

The special case in which the utility of an individual is a function only of the mean and the variance of return on the portfolio held by her has been quite popular. But this MV (mean variance) rule initiated by Markowitz (1952) and Tobin (1958), which later led to the CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966), suffers from some serious drawbacks, as has been pointed out by various authors, including Borch (1969), Feldstein (1969), and Hakansson (1972). The MV rule is optimal under two possible scenarios. One is that of quadratic utility function, which, as Hicks (1962) and Arrow (1971) have pointed out, implies IARA (increasing absolute risk aversion). To overcome this problem, one may consider restrictions on the distribution of returns.

Tobin (1958) has shown that MV rule is valid if returns are normally distributed. Samuelson (1967) has pointed out the mistake in Tobin's conjecture that all two-parameter distributions are sufficient, as have Borch (1969) and Feldstein (1969). Cass and Stiglitz (1970) have argued that only the normal distribution is sufficient, which has been countered by Agnew (1971) through an example. Chamberlain (1983) has derived that a necessary and sufficient condition for the optimality of the MV rule is that distribution of asset returns is characterized by spherical symmetry; this distribution is a special case of the class of distributions that, according to Ross (1978), imply two-fund separation. Owen and Rabinovitch (1983) have also shown that the results of Tobin (1958) and Ross (1978) yields if returns have elliptical distribution, which allows for fat tails and which contains the multivariate normal as well as multivariate non-normal distributions like multivariate Cauchy, multivariate exponential, a special multivariate Student's t , and non-normal variance mixtures of multinormal distribution. Connor (1984) has pointed out that the testable implications of the mutual-fund separation theory (the general case of the standard CAPM) and a competitive version of the well-known APT (Arbitrage Pricing Theory) are empirically indistinguishable if the analyst only observes the asset prices and investors' portfolio returns. But, Lee, Reisman, and Simann (1994) have illustrated that, instead of assuming, as Connor (1984) does, that the market portfolio is a linear function of the factors, it is assumed that the residuals obtained from regressing the asset returns on the factors are elliptically distributed conditional on the factors, then there exists a multi-beta pricing result relative to the factors and the return on the market-portfolio.

Clearly, for non-symmetric distributions, use of variance is questionable, as has been pointed out by Hirshleifer (1970; pages 278-284), Mao (1970), and Markowitz (1970; pages 188-201). Mao (1970) and Markowitz (1970) have suggested semi-variance as an alternative, and Hogan and Warren (1972) have provided an algorithm to obtain the admissible set under this rule.

In search of an alternative to the MV framework, Quirk and Saposnik (1962), Fishburn (1964), Hadar and Russell (1969), and Hanoch and Levy (1969) have obtained the optimal rule for the more general class of utility functions that are increasing in returns (higher return preferred to lower one) and thus gave birth to FSD (First order Stochastic Dominance) rule. Levy and Hanoch (1970), Levy and Sarnat (1970), and Porter and Gaumnitz (1972) have empirically verified that this rule did not achieve an appreciable reduction of the choice set. To achieve this reduction, some restriction on the utility function are necessary, and the natural restriction has been that of risk-aversion. This has led to the SSD (Second order Stochastic Dominance) rule, proposed by Hanoch and Levy (1969) and Hadar and Russell (1969, 1971). Levy and Hanoch (1970), Levy and Sarnat (1970), Porter and Gaumnitz (1972) have checked empirically that SSD-rule does indeed reduce the choice set. Whitmore (1970) has applied the TSD (Third order Stochastic Dominance) rule with the additional restriction of the third derivative of the utility function being positive, which is a necessary condition for DARA.

For spherical distributions, the SSD rule reduces to the MV rule. But, otherwise, the SSD and the MV rule lead to different admissible sets. As pointed out by Quirk and Saposnik (1962), Hadar and Russell (1969), Hakansson (1971), and Hanoch and Levy (1969), some portfolio in the MV admissible set may not be in the SSD admissible-set and, conversely, a portfolio in the SSD admissible-set may not be picked up by the MV criterion.

Thus, it is possible that a portfolio that would be chosen by a risk-averse investor may be considered unacceptable by the MV criterion! Bawa (1975) has shown that, for the entire class of distribution functions and for DARA utility, TSD rule is optimal when distributions have equal means and is sufficient when means are unequal. But, if there are no restrictions on the return distribution, no rule is necessary and sufficient; TSD rule may be used as a best approximation in such cases. Bawa (1975) has also proved that TSD also implies dominance under Mean-LPM₂ (Mean - Lower Partial Moment of order 2) rule. So, the M-LPM₂ (Mean-LPM₂) rule can be used as a reasonable approximation of the optimal rule for the entire class of return distribution. Risk from the LPM perspective is measured by taking into account only those states in which return on an asset is below a pre-specified "target rate", like the risk-free rate, and capturing the extent to which it is below. And this is desirable, since, whereas mean-variance framework assumes that investors measure risk based on both upward and downward deviations, in reality, risk actually perceived by individuals is the downside risk. Bawa and Lindenberg (1977) have developed a CAPM using the M-LPM₂ framework and showed that, when return distributions are normal, Student-t, or stable (with some constraints), their CAPM reduced to the traditional CAPM. The appeal of their framework comes from the fact that it has the MV rule as a special case under appropriate distributional assumption. But, its most attractive property is that it generates the LPM-based CAPM, a risk-return (or asset-pricing) relationship that looks identical to the CAPM, with the standard-beta substituted by the LPM-beta. Whereas Bawa (1975) has specified the riskfree-rate as the target rate, Harlow and Rao (1989) have extended this by developing a generalized Mean-LPM (MLPM) equilibrium model consistent with any target rate. Various risk-measures are special cases of this valuation framework, and, in particular, the Bawa-Lindenberg (1977) model and the Hogan and Warren (1974) semi-variance model.

It is therefore imperative to develop and apply risk-adjusted portfolio-evaluation measures based on the LPM-based CAPM (LPM-CAPM). Kochman (1999) has, in fact, recognized the importance of using downside-risk measures in portfolio-evaluation. In this paper, we develop such measures; as expected, these new measures look similar to the traditional ones mentioned above. We also provide a new way to evaluate the performance of a portfolio, which is similar to the M^2 [Modigliani-Modigliani (1997)] approach, but differs from it in an important way. We combine the portfolio to be evaluated with the risk-free asset, so that the new pseudo-portfolio has the same mean as the market-portfolio (not the same risk as M^2 does). Now that these two portfolios are on the same return footing, we compare their risks. The beauty of this approach is that we can look at more than one measure of risk simultaneously, without having to adjust the weights in the pseudo-portfolio. Using various measures of risk, we can compare the risk of the pseudo-portfolio (representing the portfolio under consideration) to that of the market portfolio and observe how it varies with the different risk-measures. We can also compare the weighted-average-risks that may be useful in certain circumstances, like when half of a portfolio-manager's clients want to look at the total risk (as they put all their money solely with this manager), while the other half looks only at the beta-risk. As we mentioned earlier, we would have two different betas, one based on the mean-variance CAPM and the other on the LPM-CAPM.

We use our new measures and approach to compare different portfolios in the Asia-Pacific area and to compare their performances with USA portfolios. Some interesting work has already been done in this area by Marchand (1990) and Montagu-Pollock and Murphy (1990). Marchand's findings that Asia-Pacific portfolios generally did well in bull-markets and poorly in bear-markets is of significance to us, as the LPM risk-measure focuses on similar trends, though generally with respect to a risk-free index.

Comparison with US markets is of interest in particular because some [see, for example, Du Bois (1998)] have reported that certain emerging-market-indices have occasionally outperformed US portfolios. Our main contribution in this arena would be to inculcate some new performance-measures and to evaluate portfolios from a new perspective.

In Section 1, we present the mean-equivalence approach of portfolio evaluation. In the next, we define LPM and characterize the LPM-CAPM. Section 3 develops portfolio-performance measures based on the LPM-CAPM. In Section 4, we describe the data and methodology. Section 5 presents the empirical results, and Section 6 concludes.

1. The Mean-Equivalence approach of portfolio evaluation

While the traditional measures by Treynor (1965) and Sharpe (1966) focuses on the excess-return (return in excess of the risk-free rate, which is usually called the risk-premium) to risk ratio, the one by Jensen (1968) is based on the abnormal return (return in excess of the RRR). Modigliani and Modigliani (1997) provide a different way to look at the problem: by combining the portfolio under evaluation with the riskfree asset to create a portfolio having the same standard-deviation as the market portfolio and comparing their returns. Here, we offer a simple alternative.

We suggest that the portfolio to be evaluated, call it P, is combined with the riskfree asset, call it RF, to create a pseudo-portfolio, PP, which has the same mean as the market portfolio, M, based on a broad-based index. Thus,

$$E(R_{PP}) = W_P E(R_P) + (1 - W_P) R_{RF} = E(R_M)$$

where R_{RF} is the risk-free rate, $E(R_P)$ is the expected return on P, $E(R_M)$ is the expected return on M, and W_P is the fraction of PP invested in P.

Obviously then,

$$W_P = \frac{E(R_M) - R_{RF}}{E(R_P) - R_{RF}}$$

Now, since the two portfolios PP and M have the same return, their risks may be compared. If the risk of PP is "lower than / equal to / higher than" the risk of M, then PP is "better than / as good as / worse than" M. Since, RF would have zero risk from almost any perspective of risk, PP's risk would be $W_P \times$ Risk of P, which should be compared to Risk of M.

If one uses traditional approaches, one has to decide *a priori* whether one wants to focus on the total risk or the non-diversifiable risk of the portfolio. If one changes the desired risk measure, the performance evaluation would change. Whereas the Sharpe (1966) and M^2 by Modigliani and Modigliani (1997) measures take the total risk of P as the relevant risk, Treynor (1965) and Jensen (1968) take the non-diversifiable or beta risk as the proper risk measure.

But, in our approach, we do not have to be confined to one risk measure. In fact, we can present all the possible risk-measures of the two portfolios, PP and M, and let people compare them on the basis of more than one risk measures. They can also use a weighted-average of more than one risk measures. For instance, suppose that W fraction of investors holding P have invested only in P, while the other fraction has invested in a variety of portfolios including P. In that case, W fraction would look at P's total risk, while (1-W) fraction would consider P's non-diversifiable risk as the relevant risk.

In that case, for comparison purposes, P's relevant risk should be given as follows:

$$\text{Weighted Risk} = [W \times \text{Total Risk of P}] + [(1 - W) \times \text{Non-diversifiable Risk of P}]$$

So, PP's risk should be taken as follows

$$\text{Risk}_{PP} = W_P \times \{ [W \times \text{Total Risk of P}] + [(1 - W) \times \text{Non-diversifiable Risk of P}] \}$$

And, it should be compared to

$$\text{Risk}_M = [W \times \text{Total Risk of M}] + [(1 - W) \times \text{Non-diversifiable Risk of M}]$$

Till today, portfolio performance evaluation has almost exclusively depended on the MV framework, which takes variance or standard-deviation as the measure of total risk and beta as the measure of non-diversifiable risk. In the next section, we discuss the LPM measures of risk, which, for reasons discussed earlier, is a very good alternative to variance.

2. LPM and the LPM-CAPM

Bawa (1975) has argued that MLPM model, based on downside risk measures, is more general than the traditional MV model, which requires restriction on utility functions or the return distribution. For arbitrary distributions, it requires the evaluation of the MLPM functional for all possible target rates. But, for the special situation of the LS (location-scale) distributions, whose univariate case has been given by Bawa (1975) and the multivariate case has been discussed in Owen and Rabinovitch (1983), the task reduces to the computation of LPM about a single point. As we have mentioned earlier, this LS class includes the normal, the Student's t (with the same degree of freedom), the stable (with the same characteristic component and the same skewness parameter, which may be non-zero) distributions.

The MLPM analysis is valid for a very general set of utility functions. To be specific, let us recall the definition of LPM, which is reproduced below.

$$LPM_n(t;F) \equiv \int_a^t (t-y)^n dF(y)$$

where, a, the lower bound may be $-\infty$ for some return distributions.

Here, the LPM is evaluated at a point t, which is pre-specified. This "t" is called the target-rate of return. This may be the risk-free rate, the return on an index, or some other benchmark.

Let us also recall that u(y) represents the utility function. As Bawa and Lindernberg (1977) have stated, for risk-averse utility-functions ($u' > 0$, $u'' < 0$), a portfolio with a return distribution F is preferred to another portfolio with return distribution G, iff

$$LPM_1(t;F) \leq LPM_1(t;G) \quad \forall t \in R \text{ and } < \text{ for some } t \text{ (where } R \text{ denotes the domain of the random returns).}$$

For risk-averse utility functions with skewness preference ($u' > 0$, $u'' < 0$, $u''' > 0$), F is preferred to G iff

$$LPM_2(t;F) \leq LPM_2(t;G) \quad \forall t \in R \text{ and } < \text{ for some } t, \text{ and } F \text{ has at least as high a mean as } G.$$

Thus, as Harlow and Rao (1989) have pointed out, the HARA class is consistent with LPM_1 , while the DARA class is consistent with LPM_2 , since DARA implies $u''' > 0$.

Bawa and Lindenberg (1977) have developed an asset-pricing model analogous to CAPM. They have derived that, for risk-averse investors, the equilibrium expected rate-of-return on an asset j is given as follows:

$$E(R_j) = R_{RF} + \beta_j^{LPM} [E(R_M) - R_{RF}]$$

where β_j^{LPM} is the beta of asset j in the LPM_1 framework and is given as follows:

$$\beta_j^{LPM} = \frac{CLPM_1(t; j, M)}{LPM_1(t; M)}$$

where $LPM_1(t; M)$, following a slight change in notation, denotes the LPM_1 of the market portfolio M, and $CLPM_1(t; j, M)$ is the Co- LPM_1 between portfolio j and the market portfolio M, as defined below:

$$CLPM_1(t; j, M) = \int_{R_M=-\infty}^t \int_{R_j=-\infty}^{\infty} (t-R_j) dF(R_M, R_j)$$

$$LPM_1(t; M) \equiv \int_{-\infty}^t (t-R_M) dF(R_M)$$

For the risk-averse individual with skewness preference, Bawa and Lindenberg (1977) and Harlow and Rao (1989) have shown that the asset-pricing model can be presented as follows:

$$E(R_j) = R_{RF} + \beta_j^{LPM_2} [E(R_M) - R_{RF}]$$

Here, $\beta_j^{LPM_2}$ is the beta of asset j in the LPM_2 framework and is given as follows:

$$\beta_j^{LPM_2} = \frac{CLPM_2(t; j, M)}{LPM_2(t; M)}$$

where $LPM_2(M)$ is the LPM_2 of the market portfolio, and $CLPM_2(j, M)$ is the Co- LPM_2 between portfolio j and the market portfolio as defined below:

$$CLPM_2(t; j, M) = \int_{R_M = -\infty}^t \int_{R_j = -\infty}^{\infty} (t - R_M) (t - R_j) dF(R_M, R_j)$$

$$LPM_2(t; M) \equiv \int_{-\infty}^t (t - R_M)^2 dF(R_M)$$

Bawa and Lindenberg (1977) have shown that, for risk-averse utility functions, the optimal portfolio choice in a MLPM framework admits separation between the risk-free asset and the market portfolio, when the LPM is calculated as deviation below the risk-free rate R_{RF} as the target rate. Harlow and Rao (1989), of course, have shown that this separation yields for any target rate, if the return distribution belongs to the location-scale class. In fact, empirical tests by Harlow and Rao (1989), which has found it hard to reject MLPM model while rejecting CAPM, has highlighted that the target-rate appears to be related more to equity-market mean returns than the riskfree rate.

As we see above, in the LPM framework, the market portfolio is considered to have risk only if the return on the market has some chance of falling below the pre-specified target rate. As in the CAPM framework, the risk of an individual security is measured by its contribution to M , the market portfolio. But, here, a particular security is said to contribute to M 's risk when its return, as well as M 's return, are below the target rate. On the other had, when the market return is below the target-rate, but the return on an asset is above the target-rate, the security's contribution to M 's risk is negative, as it reduces the risk of the M and is thus eligible for a reduction in its risk-premium. If the return on M is above the target-rate, all assets are considered riskless, independent of whether they earn above or below the target-rate. The risk of an asset across all states is the net of its positive and negative contribution to the risk of M .

3. Evaluation of Portfolio Performance: LPM-CAPM-based Measures

Now, one can develop LPM-CAPM based measures, analogous to the ones developed by Treynor (1965), Sharpe (1966), and Jensen (1968).

If we want to focus on the total risk, as done by Sharpe (1966), then we should look at LPM-based-Sharpe (LPM-S) measure.

Risk-averse individuals would look at

$$\frac{R_P - R_{RF}}{LPM_i(t; P)}$$

for each portfolio P, including the market portfolio M, and then compare them.

Risk-averse individuals with skewness preference would, however, focus on the following.

$$\frac{R_P - R_{RF}}{LPM_b(t; P)}$$

On the other hand, if we want to focus on the non-diversifiable or beta risk, then we need to compute the LPM-based Treynor (LPM-T) measure, which, for risk-averse individuals is

$$\frac{R_P - R_{RF}}{\beta_P^{LPM}}$$

and, for risk-averse individuals with skewness preference is

$$\frac{R_P - R_{RF}}{\beta_P^{LPM2}}$$

Similarly, the LPM-based Jensen's measure (LPM-J) should calculate the alpha for risk-averse individuals as follows,

$$\alpha_P = R_P - [R_{RF} + \beta_P^{LPM} \{E(R_M) - R_{RF}\}].$$

And for risk-averse individuals with skewness preference as follows

$$\alpha_P = R_P - [R_{RF} + \beta_P^{LPM2} \{E(R_M) - R_{RF}\}]$$

To compute LPM in all the measures above, we need to consider different important target-rate "t". The two prime candidates for "t" are, of course, a risk-free rate (rate on a treasury security with maturity of N days, where N typically should be one month, three months, six months, or one year) and the rate on an equity-index (like S&P-500 or Willshire 5000 in USA). Though different "t"s would lead to different conclusions, it is imperative that we look at them so as to appreciate the impact of the choice of "t" on portfolio rankings.

4. Data

We decided to take compounding intervals for obtaining the data, as many portfolio-managers are evaluated on a quarterly basis and many of them set their targets on a quarterly basis. Therefore, we obtained the required quarterly data for two sets of six-year periods: 1994-1999 and 1996-2001. We needed data on three variables: (1) Risk-free rate, (2) Return on Market Portfolio, (3) Return on Portfolios (which we wanted to evaluate). For testing robustness of our model, we had two approaches for the risk-free rate: for 94-99, we chose the borrowing-rate as measured by the call-money-rate and the lending-rate as measured by the actual return on the 90-day treasury-bill to match with our quarterly returns, while for 96-01, we chose only the 90-day treasury rate. S&P-500 was chosen as the market-portfolio and again two approaches were followed: for 94-99, actual quarterly-return on it were used, while, for 96-01, we used the percentage change in price. The risk-free and the market return were obtained from Bloomberg Data Base.

Return on portfolios (these are all reported in \$-return terms by the databases) were obtained from the Morningstar Database. For this, we had two sets: US Mutual Funds and Non-US MFs. For US funds, we made use of the 3x3 matrix that is used to breakdown the funds by their market-capitalization (low, mid, high) and by their potential for growth. For the 94-99 period, we used capitalization in 2000, and for the 96-01 period, the capitalization in 2002. For each of the 9 segments, we selected the firm that is ranked 33rd and the one that is ranked 66th in percentile-ranking. For non-US funds, we chose some Japanese, Korean, Indian, Chinese, Thai, and Malaysian funds; the criterion was that they must have the data for the required quarters. We then rejected those funds where the average return over the 24 quarters was less than the average risk-free rate for the corresponding period. We thus ended up with 24 firms for the 94-99 period and 22 for the 96-01 period.

5. Methodology

We first computed the average risk-free rate for the 24 quarters and the corresponding return on the S&P-500. For 94-99, we had both borrowing and lending rates and the return on S&P-500, while, for 96-01, we obtained the three-month treasury-bill yield and the percentage-change in S&P-500 level. Then, we obtained the standard-deviation (σ), LPM-1, and LPM-2 of the market-portfolio and measured its performance using the Treynor, Sharpe, Jensen, LPM-Treynor, LPM-Sharpe (using both LPM-1 and LPM-2 as the total-risk measure), and LPM-Jensen. We also computed the risk of the market-portfolio, assuming that a certain percentage of investor focus on diversifiable-risk and the other on total risk.

Then, for each of the funds, we computed the above risk-measures as well as beta and LPM-beta. We then analyzed their performance, using the same measures we used for the market portfolio. After this, following our new approach, we determined the weight on the portfolio that makes a pseudo-portfolio consisting of the portfolio and the risk-free asset get the same return as the market portfolio. Thereafter, we computed weighted risk measures of this pseudo-portfolio and compared it to that of the market. We also compared the risk of the market portfolio and this pseudo-portfolio assuming a certain percentage of investor focus on diversifiable-risk and the other on total risk; these results are, however, not reported here.

6. Findings

We found that almost half the funds had LPM-beta lower than the CAPM-beta, which tells us that, half the time, we overestimate the risk of the mutual-funds by using the CAPM-beta instead of LPM-beta, which is the correct risk-measure when return distributions are not normal, which is usually the case.

But to get a feel for the implication of the LPM-measure for portfolio-evaluation, let us focus on some particular funds. The following table gives the performance statistics of S&P-500 and two funds for each quarter. Data are in percentages.

<u>Statistic</u>	<u>S&P-500</u>	<u>ALMRX</u>	<u>KIF</u>	<u>S&P 500</u>	<u>IFN</u>	<u>QBNAX</u>
Period	94-99	94-99	94-99	96-01	96-01	96-01
Mean	5.65	7.11	5.39	2.94	2.63	3.45
Sigma	7.06	11.47	35.03	9.22	19.02	19.87
Beta	1.00	1.35	2.67	1.00	0.85	1.77
LPM-1	1.05	2.16	8.88	2.84	6.69	6.76
LPM-2	8.37	18.70	302.51	32.50	112.74	129.99
LPM-Beta	1.00	1.11	0.63	1.00	0.71	1.78
Treynor	4.40	4.33	1.55	1.71	1.64	1.91
Sharpe	0.62	0.51	0.12	0.19	0.07	0.16
Jensen	0.00	-0.10	-7.62	0.00	-0.06	0.24
LPM-Treynor	4.40	5.29	6.53	1.71	1.99	2.01
LPM-1 Sharpe	4.19	2.71	0.47	0.60	0.21	0.46
LPM-2 Sharpe	0.53	0.31	0.01	0.05	0.01	0.03
LPM-Jensen	0.00	0.99	1.35	0.00	0.20	0.33
Weighted Sigma	7.06	8.61	37.48	9.22	23.13	10.49
Weighted Beta	1.00	1.02	2.86	1.00	1.04	0.89
Weighted LPM-1	1.05	1.62	9.49	2.84	8.14	3.69
Weighted LPM-2	8.37	14.04	323.44	32.50	137.14	50.17
Weighted LPM-beta	1.00	0.83	0.68	1.00	0.86	0.85

The data reveals some interesting findings. First of all, using our LPM measures, in 94-99 period, some funds did better than S&P, while some did worse. But, for 96-01 period, most funds outperformed the market using the same measures. But, the portfolio performance evaluation is where our focus is. Let us take ALMRX from the 94-99 period. As per all the three traditional measures, it was beaten by the market. That also holds when we use our LPM based Sharpe measures. But, looking at LPM-Treynor and LPM-Jensen does give the opposite feeling. When we compare the weighted-risk of the pseudo portfolio with that of the market, most of these impressions are borne out: the weighted LPM-beta of the pseudo-portfolio, 0.83, is strictly lower than the market LPM-beta of 1, though all other weighted-risk measures of the pseudo-portfolio is higher. This tells us the following. If we combine ALMRX with the risk-free asset to create a portfolio with the same return as the market, it would have higher sigma, beta, LPM-1, and LPM-2, but lower LPM-Beta. If LPM-beta is the correct risk-measure – and it should be if returns are not normally distributed – this fund has actually beaten the market. The story for the Korean fund, KIF, is the same. And, it is repeated for IFN for the period 96-01. But, the story of QBNAX is somewhat different. Using sigma-type measures (Sharpe, LPM-1 Sharpe, LPM-2 Sharpe,) it does worse than the market portfolio, while, if we use the beta-type measures (Treynor, LPM Treynor, Jensen, LPM Jensen), it does worse. Our weighting method highlights this by giving both a lower weighted-beta and lower weighted LPM-beta.

In some cases (not detailed here), we got negative LPM-betas. This is too good, as it implies that the risk of the asset is negative, though it earns more than the risk-free rate. This superior performance is also corroborated by the lower weighted-LPM-beta of its pseudo-portfolio compared to the market portfolio.

But, it is interesting to note that, in almost all cases, the overall risk of the mutual-funds, as measured by sigma-type measures (sigma, LPM-1, LPM-2) is higher than the market. This does indeed corroborate the well-known belief that the market portfolio is the best diversified broad-based portfolio.

References

Agnew, R.A. "Counter-examples to an assertion concerning the normal distribution and a new stochastic price fluctuation model", *Review of Economic Studies*, 38, 1971, 381-383

Arrow, K.J., Essays in the Theory of Risk Bearing, 1971, Markham, Chicago, Illinois

Bawa, V. "Optimal Rules for Ordering Uncertain Prospects", *Journal of Financial Economics*, 2, March 1975, 95-121

Bawa, V. and E. Lindenberg "Capital Market Equilibrium in a Mean, Lower Partial Moment Framework", *Journal of Financial Economics*, 5, November 1977, 189-200

Borch, K. "A note on uncertainty and indifference curves", *Review of Economic Studies*, 36, 1969, 1-4

Cass, D. and J. Stiglitz "The structure of investor preferences and asset returns, and separability in portfolio theory: A contribution to the pure theory of mutual funds", *Journal of Economic Theory*, 2, 1970, 122-160

Chamberlain, G. "A Characterization of the Distributions That Imply Mean-Variance Utility Functions", *Journal of Economic Theory*, 29, February 1983, 185-201

Connor, G. "A Unified Beta Pricing Theory", *Journal of Economic Theory*, 34, October 1984, 13-31

Du Bois, P. "International trader: Sixteen of the world's bourses outperformed U.S. in first quarter", *Barron's*, April 6 1998, 78, MW7-MW9

Feldstein, M.S. "Mean-variance analysis in the theory of liquidity preference and portfolio selection", *Review of Economic Studies*, 36, 1969, 5-12

Fishburn, P.C., Decision and Value Theory, 1964, Wiley, New York

Hadar, J. and W.R. Russell "Rules for ordering uncertain prospects", *American Economic Review*, 59, 1969, 25-34

Hadar, J. and W.R. Russell "Stochastic dominance and diversification", *Journal of Economic Theory*, 3, 1971, 288-305

Hakansson, N. "Capital growth and the mean-variance approach to portfolio selection", *Journal of Financial and Quantitative Analysis*, 6, 1971, 517-557

Hakansson, N. "Mean-variance analysis in a finite world", *Journal of Financial and Quantitative Analysis*, 5, 1972, 1873-1880

Hanoch, G. and H. Levy "The efficiency analysis of choices involving risk", *Review of Economic Studies*, 36, 1969, 335-346

Harlow, W.V. and R.K.S. Rao "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence", *Journal of Financial and Quantitative Analysis*, 24, September 1989, 285-311

Hirshleifer, J., Investment, Interest, and Capital, 1970, Prentice Halls, Englewood Cliffs, N.J.

Hogan, W. and J. Warren "Computation of the Efficient Boundary in the E-S Portfolio Selection Model", *Journal of Financial and Quantitative Analysis*, 7, September 1972, 1881-1896

Jensen, M. "The Performance of Mutual Funds in the Period 1945-1964", *Journal of Finance*, 23, May 1968, 389-416

Kochman, L.M. "Portfolio Evaluation, Downside Risk and an Anomaly", *American Business Review*, 17, 1 June 1999, 53-58

Lee, C.F., H. Reisman, and Y. Simaan "A note on the generalized multibeta CAPM", *Mathematical Finance*, 4, January 1994, 67-68

Levy, H. and G. Hanoch "Relative effectiveness of efficiency criteria for portfolio selection", *Journal of Financial and Quantitative Analysis*, 5, 1970, 63-76

Levy, H. and M. Sarnat "Alternative efficiency criteria: An empirical analysis", *Journal of Finance*, 25, 1970, 1153-1158

Lintner, J. "Security prices, risk and maximal gain from diversification", *Journal of Finance*, 30, 1965, 587-615

Mao, J.C.T. "Models of capital budgeting, E-V vs. E-S", *Journal of Financial and Quantitative Analysis*, 4, 1970, 657-675

Marchand, C. "Investment Strategy: Risky Attractions", *Far Eastern Economic Review*, 147, Jan 4 1990, 39-40

Markowitz, H. "Portfolio Selection", *Journal of Finance*, 7, 1952, 77-91

Markowitz, H., Portfolio Selection: Efficient diversification of investments, 1970, Wiley, New York

Modigliani, F. and Leah Modigliani "Risk-Adjusted Performance", *Journal of Portfolio Management*, 23, Winter 1997, 45-54

Montagu-Pollock, M. and D. Murphy "New Perspectives on Mutual Funds", *Asian Business*, 26, October 1990, 56-65,

- Mossin, J. "Equilibrium in a capital market", *Econometrica*, 34, 1966, 768-783
- Owen, J. and R. Rabinovitch "On the Class of Elliptical Distributions and their Applications to the Theory of Portfolio Choice", *Journal of Finance*, 38, June 1983, 745-752
- Porter, R.B. and J.E. Gaumnitz "Stochastic dominance vs. mean-variance portfolio analysis: An empirical evaluation", *American Economic Review*, 62, 1972, 438-446
- Pratt, J.W. "Risk aversion in the small and in the large", *Econometrica*, 32, 1964, 122-136
- Qurik, J.P. and R. Saposnik "Admissibility and measurable utility functions", *Review of Economic Studies*, 29, 1962, 140-146
- Ross, S.A. "Mutual fund separation in financial theory - the separating distributions", *Journal of Economic Theory*, 17, 1978, 254-286
- Samuelson, P.A. "General proof that diversification pays", *Journal of Financial and Quantitative Analysis*, 2, 1967, 1-13
- Sharpe, W.F. "Capital asset prices: A theory of market equilibrium under conditions of risk", *Journal of Finance*, 29, 1964, 425-442
- Sharpe, W.F. "Mutual Fund Performance", *Journal of Business*, 39, January 1966, 119-138
- Stiglitz, J.E. "Review of some aspects of theory of risk-bearing by K.J. Arrow", *Econometrica*, 38, 1970
- Tobin, J. "Liquidity preference as behavior towards risk", *Review of Economic Studies*, 25, 1958, 65-86
- Treynor, J.L. "How to Rate Management Investment Funds", *Harvard Business Review*, 43, January-February 1965, 63-75
- Von Neumann, J. and O. Morgenstern, Theory of Games and Economic Behavior, 1967, Wiley, New York
- Whitmore, G.A. "Third order stochastic dominance", *American Economic Review*, 50, 1970, 457-459